5. *The Plancherel Formula for the Universal Covering Group of De Sitter Group*

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In his recent paper [3], R. Takahashi conjectured the explicit Plancherel formula for the universal covering group of De Sitter group.

The purpose of this paper is to prove that this formula is actually the Plancherel formula of the group.

The method in the present paper can be applied for other groups. For simplicity, however, we confine our considerations only to the above mentioned group.

1. Let G be the universal covering group of De Sitter group realized in [3].

We define three one-parameter subgroups whose generic elements are;

$$m_\varphi=\begin{pmatrix} e^{i\varphi/2} & 0 \\ 0 & e^{i\varphi/2}\end{pmatrix},\qquad a_t=\begin{pmatrix}\mathrm{ch}t/2 & \mathrm{sh}t/2\\ \mathrm{sh}t/2 & \mathrm{ch}t/2\end{pmatrix},\qquad u_\theta=\begin{pmatrix} e^{i\theta/2} & 0\\ 0 & e^{-i\theta/2}\end{pmatrix}$$

respectively and denote by H_0, H_1, H_2 the left invariant infinitesimal transformations defined by these subgroups. Put

$$A_1=\{a_t m_\varphi;\ t,\varphi\in \boldsymbol{R}\},\qquad A_2=\{u_\theta m_\varphi;\ \theta,\varphi\in\boldsymbol{R}\}\ .$$

Then A_1 and A_2 are the non conjugate Cartan subgroups of G (see [1 (b)]). Every Cartan subgroup of G is conjugate with either A_1 or A_2 (see [2]). We put $G_k=\bigcup_{g\in G} gA_kg^{-1}$ $(k=1, 2)$.

Let $U_{n,\nu}$ and $T_{n,p}$ be the characters of the representations $U^{n,3/2+i\nu}$ and $T^{n,0,p}\oplus T^{0,n,p}$ defined in [3] respectively, then there are locally summable functions $\chi^{(1)}_{n,\nu}, \chi^{(2)}_{n,p}$ on G such that

$$U_{n,\nu}(f)=\int_G f(g)\chi^{(1)}_{n,\nu}(g)dg,\qquad T_{n,p}(f)=\int_G f(g)\chi^{(2)}_{n,p}(g)dg,$$

where dg is the Haar measure on G (cf. [1(f)]). Let $\mathfrak{g}, \mathfrak{h}_1, \mathfrak{h}_2$ be the Lie algebras of G, A_1, A_2 respectively.

There exists a Cartan involution θ of $\mathfrak{g}$ such that $\theta\mathfrak{h}_k=\mathfrak{h}_k$ $(k=1, 2)$. Let $\mathfrak{g}=\mathfrak{k}+\mathfrak{p}$ be the corresponding Cartan decomposition of $\mathfrak{g}$.

We can select compatible orderings in the dual spaces of $\mathfrak{h}_k\cap\mathfrak{p}$ and $\mathfrak{h}_k\cap\mathfrak{p}+i\mathfrak{h}_k\cap\mathfrak{k}$ (see [1(d)]). Let P_k be all positive roots in this order. Put $P_k^+=\{\alpha\in P_k;\ \alpha(\mathfrak{h}_k\cap\mathfrak{p})\neq\{0\}\}$, then $P_k-P_k^+$ is the disjoint sum of the set P_k^0 of all non compact positive roots and the set P_k^- of all compact positive roots (see 1(b)). We put

$$\Delta_k(\exp H)=\Big|\prod_{\alpha\in P_k^+}(e^{\alpha(H)/2}-e^{-\alpha(H)/2})\Big|\prod_{\alpha\in P_k^0\cup P_k^-}(e^{\alpha(H)/2}-e^{-\alpha(H)/2})$$