34. Some Applications of the Functional-Representations of Normal Operators in Hilbert Spaces. XV

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Let $N_j, D_j (j=1, 2, 3, \cdots, n)$, $\{\lambda_\nu\}_{\nu=1,2,3,\cdots}$, $f_{1\alpha}$, $f_{2\alpha}$, $f'_{1\alpha}$, $f'_{2\alpha}$, $g_{j\beta}$, $g'_{j\beta}$, and $T(\lambda)$ be the same notations as those defined in Part XIII (cf. Proc. Japan Acad., Vol. 40, No. 7, 492–493 (1964)), and let $R(\lambda)$ be the ordinary part of $T(\lambda)$. Then

$$T(\lambda)=R(\lambda)+\sum_{\alpha=1}^{m}((\lambda I-N_1)^{-\alpha}(f_{1\alpha}+f_{2\alpha}),$$
$$(f'_{1\alpha}+f'_{2\alpha}))+\sum_{j=2}^{n}\sum_{\beta=1}^{k_j}((\lambda I-N_j)^{-\beta}g_{j\beta}, g'_{j\beta}),$$

and $T(\lambda)$ possesses the properties (i), (ii), and (iii) described in Part XIII. Analytically speaking, the first principal part of $T(\lambda)$ is given by

$$\sum_{\alpha=1}^{m}((\lambda I-N_1)^{-\alpha}f_{1\alpha}, f'_{1\alpha})=\sum_{\alpha=1}^{m}\sum_{\nu=1}^{\infty}\frac{c_\alpha^{(\nu)}}{(\lambda-\lambda_\nu)^\alpha},$$

where if $\lambda_1=\lambda_2=\cdots=\lambda_{m_1}$, $\lambda_{m_1+1}=\lambda_{m_1+2}=\cdots=\lambda_{m_2}$, and so on, then $\sum_{\nu=1}^{\infty}\frac{c_\alpha^{(\nu)}}{(\lambda-\lambda_\nu)^\alpha}$ means the sum

$$\frac{c_\alpha^{(1)}}{(\lambda-\lambda_1)^\alpha}+\frac{c_\alpha^{(m_1+1)}}{(\lambda-\lambda_{m_1+1})^\alpha}+\cdots=\frac{c_\alpha^{(m_1)}}{(\lambda-\lambda_{m_1})^\alpha}+\frac{c_\alpha^{(m_2)}}{(\lambda-\lambda_{m_2})^\alpha}+\cdots,$$

as will be seen by the definition of $c_\alpha^{(\nu)}$ in the above-mentioned paper; and in addition, the second principal part of $T(\lambda)$ is given by

$$\sum_{\alpha=1}^{m}((\lambda I-N_1)^{-\alpha}f_{2\alpha}, f'_{2\alpha})+\sum_{j=2}^{n}\sum_{\beta=1}^{k_j}((\lambda I-N_j)^{-\beta}g_{j\beta}, g'_{j\beta})$$
$$=\sum_{\alpha=1}^{m}\int_{\Omega\cup D_1}\frac{1}{(\lambda-z)^\alpha}d(K^{(1)}(z)f_{2\alpha}, f'_{2\alpha})+\sum_{j=2}^{n}\sum_{\beta=1}^{k_j}\int_{D_j}\frac{1}{(\lambda-z)^\beta}d(K^{(j)}(z)g_{j\beta}, g'_{j\beta}),$$

where Ω denotes the set of all those accumulation points of $\{\lambda_\nu\}_{\nu=1,2,3,\cdots}$ which do not belong to $\{\lambda_\nu\}$ itself and $\{K^{(j)}(z)\}$ is the complex spectral family associated with the bounded normal operator N_j $(j=1, 2, 3, \cdots, n)$. These facts are clear from the respective definitions of the notations $f_{1\alpha}$, $f_{2\alpha}$, $f'_{1\alpha}$, $f'_{2\alpha}$, $g_{j\beta}$, $g'_{j\beta}$, $c_\alpha^{(\nu)}$, N_j, and D_j.

Since, by definition, $\{\lambda_\nu\}$ is an arbitrarily prescribed bounded set of denumerably infinite complex numbers, we may and do suppose here that it is everywhere dense on an open rectifiable Jordan curve Γ; and as a special case, we consider the function $\hat{T}(\lambda)$ defined by

$$\text{(A)}\quad \hat{T}(\lambda)=R(\lambda)+\sum_{\alpha=1}^{m}((\lambda I-N_1)^{-\alpha}f_{1\alpha}, f'_{1\alpha})+\sum_{j=2}^{n}\sum_{\beta=1}^{k_j}((\lambda I-N_j)^{-\beta}g_{j\beta}, g'_{j\beta}).$$

Then it is obvious that every λ_ν is a pole of $\hat{T}(\lambda)$ in the sense of the