25. On the Covering Dimension of Certain Product Spaces

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In our previous paper [5], we have proved: *If a product space* $X\times Y$ *of a space* X *with a separable metric space* Y *is countably paracompact and normal, then*

$$\dim(X\times Y)\leqq \dim X+\dim Y.$$

Here $\dim X$ means the covering dimension of X.

In the present paper, we shall establish that if X is a normal P-space [I] the above inequality holds for any metric space Y with an open basis which is a countable union of star-finite systems, even if Y is not separable. Here, *a topological space* X *is called a* P-*space if for any set* Ω *of indices and for any family* $\{G(\alpha_1, \alpha_2, \cdots, \alpha_i)\mid \alpha_\nu\in\Omega;\ i=1, 2, \cdots\}$ *of open subsets of* X *such that* $G(\alpha_1, \cdots, \alpha_i)\subset G(\alpha_1, \cdots, \alpha_i, \alpha_{i+1})$ *for* $\alpha_\nu\in\Omega$ *and* $i=1, 2, \cdots$, *there is a family* $\{F(\alpha_1, \cdots, \alpha_i)\mid \alpha_\nu\in\Omega;\ i=1, 2, \cdots\}$ *of closed subsets of* X *such that* (a) $F(\alpha_1, \cdots, \alpha_i)\subset G(\alpha_1, \cdots, \alpha_i)$ *for* $\alpha_\nu\in\Omega(\nu=1, \cdots, i)$ *and* (b) $X=\bigcup_{i=1}^{\infty} F(\alpha_1, \cdots, \alpha_i)$ *provided that* $X=\bigcup_{i=1}^{\infty} G(\alpha_1, \cdots, \alpha_i)$.

This concept of P-spaces which is weaker than perfect normality and somewhat stronger than countable paracompactness was introduced by K. Morita [I] in his study on the normality of product spaces, and it was established by him that X is a normal P-space if and only if $X\times Y$ is normal for any metric space Y. Thus our assumption imposed upon X may be said to be reasonable. It is to be noted that every separable metric space has always an open basis which is star-finite.

Theorem 1 has been already proved by K. Morita in his unpublished paper, but in this paper we shall give our proof for the sake of completeness and for its own interest.

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1. The following Lemma has been already presented in [5] with more general form.

Lemma. *If* $\dim Y=0$ *for a metric space* Y, *there are a countable number of open coverings* $V_i=\{V_{i\alpha}\mid \alpha\in\Omega_i\}$ $(i=1, 2, \cdots)$ *of* Y *such that* (a) $V_{i\alpha}$ *is open and closed for any* i *and* α, (b) $V_{i\alpha}\cap V_{i\beta}=\phi$ *provided* $\alpha\neq\beta$, (c) $\bigcup_i V_i$ *is an open basis of* Y.