47. On the Convergence Theorem for Star-shaped Sets in E^n

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Introduction. It is well known, as Blaschke convergence theorem, that a uniformly bounded infinite collection of closed convex sets in a finite dimensional Minkowski space contains a sequence which converges to a non-empty compact convex set. The convergence problem for star-shaped sets seems open up to-day (cf. [1]).

In this paper, modifying F. A. Valentine's proof of the Blaschke convergence theorem in [1], we prove a convergence theorem for star-shaped sets in the n-dimensional euclidean space E^n. In the case of E^3, Z. A. Melzak's result [2] is known.

1. Notations and lemmas. In the following, we consider sets in the n-dimensional euclidean space E^n only.

Let S be a star-shaped set relative to a point p. Then the closure of S, denoted by clS, is a star-shaped set relative to the point p. If $\{S^\alpha;\ \alpha\in \text{index set}\}$ is a finite or an infinite collection of star-shaped sets relative to a point p, then $\bigcup_\alpha S^\alpha$ and $\bigcap_\alpha S^\alpha$ are star-shaped relative to the point p.

An *ε-parallel set* A_ε of a set A is defined by

$$A_\varepsilon \equiv \bigcup_{a\in A} K(a,\ \varepsilon),\ (0\leqq\varepsilon,\ \varepsilon\in \text{reals}),$$

where $K(a,\ \varepsilon)$ denotes the solid sphere with center a and radius ε. The distance between the two points x and y is denoted by $d(x,\ y)$.

Lemma 1. $(A_\rho)_\sigma\subset A_{\rho+\sigma}$.

Proof. Let x be a point in $(A_\rho)_\sigma$. Then there is a point $y\in A_\rho$ such that $d(x,\ y)\leqq\sigma$. Similarly there is a point $z\in A$ such that $d(y,\ z)\leqq\rho$. Hence we have

$$d(x,\ z)\leqq d(x,\ y)+d(y,\ z)=\sigma+\rho.$$

Therefore x is a point of $A_{\rho+\sigma}$.

The distance $d(A,\ B)$ between the two sets A and B is defined by

$$d(A,\ B)=\inf_{\substack{A\subset B_\rho\\ B\subset A_\rho}} \rho.$$

If A and B degenerate to two points x and y, the distance function coincides with the ordinary distance of E^n.

Lemma 2. *A collection of compact sub-sets becomes a metric space with the metric defined above.*