61. *On Linear Isotropy Group of a Riemannian Manifold*

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(Comm. by Kinjirô KUNUGI, M.J.A., April 12, 1965)

Introduction. Let M be a connected Riemannian manifold of dimension n and of class C^∞, and let M_p be the tangent space of M at p. According to the Riemannian structure a scalar product $g_p(X, Y)$ is defined for any vectors $X, Y \in M_p$. We denote by L_p the group of all linear transformations of M_p. The *infinitesimal linear isotropy group* K_p is, by definition [2], the subgroup of L_p consisting of all linear transformations of M_p which leave invariant the curvature tensor R and the successive covariant differentials ∇R, $\nabla^2 R, \cdots$ at p. We define a group A_p as a subgroup of K_p consisting of all elements of K_p which leave invariant the scalar product $g_p(X, Y)$. Let $I(M)$ be the group of isometries of M. Let H_p be the isotropy group of $I(M)$ at p, and let dH_p be the linear isotropy group of H_p. In §1, we shall investigate sufficient conditions that $dH_p = A_p$. §2 is devoted to applications of the main theorem to Riemannian globally symmetric spaces.

§1. Main theorem.

Theorem 1. *If M is a simply connected homogeneous Riemannian manifold, then $dH_p = A_p$ for each p in M.*

In order to prove this theorem, we need the following:

Lemma. *If M is an analytic complete simply connected Riemannian manifold, then $dH_p = A_p$ for each p in M.*

Proof. We have proved that $dH_p \subset A_p$ for any Riemannian manifold [3] p. 1). Take a normal coordinate system $\{x_1, \cdots, x_n\}$ at p, with coordinate neighborhood U. We may assume that $\{(\partial/\partial x_1)_p, \cdots, (\partial/\partial x_n)_p\}$ is an orthonormal base, and that U is the interior of a gedesic sphere centered at p. U has the Riemannian metric induced from M. Since M is analytic, each element $a \in A_p$ induces a local isometry $\tilde{f}$ which maps U onto itself, such that $\tilde{f}(p) = p$ and $(d\tilde{f})_p = a$ ([3] p. 2). Since M is a simply connected complete analytic Riemannian manifold, and U is a connected open subset of M, this local isometry $\tilde{f}$ can be uniquely extended to f, an isometry of M ([4] p. 256). Clearly $f(p) = p$ and $(df)_p = a$. Therefore we have $A_p \subset dH_p$.

Proof of Theorem. Since M is a Riemannian homogeneous