60. On Almost Periodic Transformations on Metric Space over Topological Semifield

By Kiyoshi ISÉKI

(Comm. by Kinjirô KUNUGI, M.J.A., April 12, 1965)

A. Edrei, P. Erdös, W. H. Gottschalk, G. A. Hedlund, and A. H. Stone have obtained interesting results on transformations on topological spaces (for references, see [5]). In this note, we shall consider some results in a metric space over a topological semifield (for related concepts, see [1] and [2]). Let X be a metric space over a topological semifield R. We denote the metric by ρ. Let f be a continuous mapping on X, i.e. $f(X)\subset X$.

We first repeat some definitions needed.

The mapping f is said to be *strongly almost periodic* if for a given neighborhood U there is a positive integer k such that every k consecutive positive integers contains an n satisfying $\rho(x, f^n(x))\in U$ for all $x\in X$.

In the definition of strongly almost periodicity, the positive integer k is independently taken for each point x of X. If k depends on each point x, we need a new definition.

A point x of X is said to be *almost periodic* under f (by W. H. Gottschalk [4]) if for a given neighborhood U, there is a positive integer k such that every k consecutive positive integers contains n satisfying $\rho(x, f^n(x))\in U$. If each point x is almost periodic under f, the mapping f is said to be *pointwise almost periodic*. For $x\in X$, the set $\bigcup_{n=-\infty}^{\infty} f^n(x)$ is called the *orbit* of x under f and the set $\bigcup_{n=0}^{\infty} f^n(x)$ is called the *semi-orbit* of x under f.

Under these concepts, we shall prove the following theorem which is formulated by P. Erdös and A. H. Stone [3].

Theorem. Let X be a totally bounded metric space over a topological semifield, and f a homeomorphism of X. If the set of all negative powers is equiuniformly continuous, then f is strongly almost periodic.

The proof is quite similar with that of Theorem III of P. Erdös and A. H. Stone [3].

To prove Theorem, we take a neighborhood U of 0 in R. Then there is a neighborhood W such that $W+W\subset U$. For W, there is a neighborhood V of 0 such that $\rho(f^{-m}(x), f^{-m}(y))\in W$ holds for x, y of $f^m(x)$ for which $\rho(x, y)\in V$. Here we can take V and W as saturated neighborhoods and $V\subset W$.