59. *Resolvent Kernels on a Martin Space*

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Let R be a Green space, M be its Martin boundary and μ be the harmonic measure on M relative to a fixed point of R. As a result of the author's previous paper [3], we can see that, if every point of M is an exit boundary point of R,

$$(1)\qquad D(u, u)=\int_M\int_M (u(\xi)-u(\eta))^2 U(\xi, \eta)\mu(d\xi)\mu(d\eta)$$

represents the Dirichlet integral on R, up to a constant, of the harmonic function with boundary value $u(\xi)$, $\xi\in M$, where $U(\xi, \eta)$ is Feller's kernel (cf. Doob [2]).

We shall apply this fact to form a system of resolvent kernels on $(R\cup M)\times R$ which dominate on $R\times R$ the resolvent kernels of a Brownian motion on R. As the generalized normal derivatives of the potentials defined by these kernels, we may have zero function on M. The construction of these kernels is our main purpose.

To this aim, we shall first define a system of operators R^α, $\alpha>0$ on $L^2(\mu)$ such that, for every $\varphi\in L^2(\mu)$, $R^\alpha\varphi$ satisfies

$$(2)\qquad D(R^\alpha\varphi, v)+2\int_M\int_M R^\alpha\varphi(\xi)U_\alpha(\xi, \eta)v(\eta)\mu(d\xi)\mu(d\eta)=2\int_M \varphi(\xi)v(\xi)\mu(d\xi)$$

for any v in a certain function class, where U_α is α-order Feller's kernel. Next, we shall prove the positivity of $R^\alpha(\alpha>0)$ and the continuity of $R^\alpha\varphi$ in a certain sense. Finally, using $\{R^\alpha, \alpha>0\}$, we shall form resolvent kernels satisfying the properties cited above.

§ 1. Positive operators $R^\alpha(\alpha>0)$ on $L^2(\mu)$.

Let $p(t, x, y)$ $t>0, x, y\in R$ be the transition function of a Brownian motion on R. Its resolvent kernel is defined by

$$G_\alpha(x, y)=\int_0^{+\infty} e^{-\alpha t}p(t, x, y)dt,\ \alpha>0,\ x, y\in R.$$

For the Martin K-function $K(x, \xi)$, put

$$K_\alpha(x, \xi)=K(x, \xi)-\alpha\int_R G_\alpha(x, y)K(y, \xi)dy, \qquad x\in R, \xi\in M, \alpha>0.$$

We call $\xi\in M$ an exit boundary point if and only if $K_\alpha(x, \xi)\neq 0$ for some $x\in R$ and $\alpha>0$. For $\xi, \eta\in M$, $\alpha>0$, $U_\alpha(\xi, \eta)=\alpha\int_R K(x, \xi)K_\alpha(x, \eta)dx$ is monotone increasing in α and we call $U(\xi, \eta)=\lim\limits_{\alpha\to+\infty} U_\alpha(\xi, \eta)$ Feller's kernel (cf. [3]).

From now on, we assume that

(A.1) almost every (μ) point of M is exit,