83. A New Interpretation of Gottschalk's Results on Almost Periodicity

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Let X be a metric space over a topological semifield R. The concept of such a metric space was introduced in M. Ya. Antonovski, V. G. Boltjanski and T. A. Sarimsakov ((1) and (2)). Let T be a topological group, and let $f(x, t)$ be a mapping from $X\times T$ to X. We often use two conventional notations $f^t(x)$, $f^x(t)$ instead of $f(x, t)$. For the family $\{f(x, t)\}$, we introduce two concepts.

Definition 1. $\{f^t(x)\}$ is said to be *equicontinuous* at x of X, if for every neighborhood U of 0 in R, there is a neighborhood V of 0 in R such that $y\in \Omega(x, V)$ implies $\rho(f^t(x), f^t(y))\in U$ for all $t\in T$.

If $\{f^t(x)\}$ is equicontinuous at each point of X, $\{f^t(x)\}$ is said to be *equicontinous* on X.

Definition 2. $\{f^t(x)\}$ is said to be *equiuniformly continuous*, if for every neighborhood U of 0 in R, there is a neighborhood V of 0 in R such that $y\in\Omega(x, V)$ implies $\rho(f^t(x), f^t(y))\in U$.

Of course ρ denotes the metric on X.

The set $f(x, T)$ is called the *orbit* of x. Following W.H. Gottschalk (3), we shall define that x of X is almost periodic.

A subset D of T is said to be *relatively dense*, if there is a compact set A of T such that each left translate of A meets D, i.e. $T=DA$. A point x of X is said to be *almost periodic*, if for every neighborhood $\Omega(x, U)$ of x there is a relatively dense set D in T such that $f(x, D)\subset\Omega(x, U)$.

Then we have the following two propositions on almost periodicity which is obtained by W.H. Gottschalk (3). We shall suppose that $f^t(x)$ is a transformation group on X.

Theorem 1. *If the family* $\{f^t(x)\}$ *is equiuniformly continuous, and the orbit of* x *is totally bounded, then* x *is almost periodic.*

Proof. For a neighborhood U of 0 in R, there is a neighborhood V of 0 in R such that $\rho(x, y)\in V$ implies $\rho(f^t(x), f^t(y))\in U$.

$f(x, T)$ is totally bounded, then we can find a finite set $t_1, t_2, \cdots, t_n$ of T such that $f(x, T)\bigcup_{i=1}^{n}\Omega(f^{t_i}(x), V)$. Hence for $t\in T$, there is at least one t_i for which $f^t(x)=f(x, t)\in\Omega(f^{t_i}(x), V)$. So we have $\rho(f^{tt_i^{-1}}(x), x)\in U$, and then $f^{tt_i^{-1}}(x)\in\Omega(x, U)$. Therefore x is almost periodic.

Theorem 2. *If the family* $\{f^t(x)\}$ *is equicontinuous at* x, $f^x(t)$