82. *Remarks on a Continuous Kernel*

By Masanori KISHI and Toshio SHIMAZAKI
Mathematical Institute and Department of Engineering Mathematics, Nagoya University

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1. **Introduction.** Choquet and Deny [1, 2] proved the following theorem: A strictly positive continuous kernel $V: C_K \to C$ satisfies the balayage principle if and only if it satisfies the domination principle. (For the notations and the definitions see Section 2.) In the present note we show that a continuous kernel $V: C \to C$ satisfies the balayage principle on any open set if and only if it satisfies the domination principle under the assumption that $V(C)$ is dense in C. In Section 4 we show that if a continuous kernel $V: C \to C$ satisfies the two conditions, the denseness of $V(C)$ in C and the complete maximum principle, then it is a continuous kernel of Hunt.

2. **Notations and definitions.** Let X be a locally compact Hausdorff space, and B denote the Borel field on X. Let $C = C(X)$ be the totality of bounded continuous real valued functions on X. C is a real Banach space with the norm $\|f\| = \sup_{x \in X} |f(x)|$. Let $C_K = C_K(X)$ be the totality of continuous real valued functions on X with compact support. Let $M = M(X)$ and $M_K = M_K(X)$ be the totalities of real Radon measures on X and of those with compact support, respectively. We denote by $C^+, \cdots$ the subsets of the above sets consisting of positive elements.

Definition 1. A mapping V of $X \times B$ into $[0, +\infty]$ is called a *kernel* on X, if it has the following properties: For any $x \in X$, the set function $V(x, e)$ of e is a positive Radon measure on X, and for any relatively compact $e \in B$, the function $V(x, e)$ of x is a locally bounded Borel function.

Given a positive Borel function f, *its potential* $Vf(x)$ is defined by

$$Vf(x) = \int f(y) V(x, dy).$$

Given a positive Radon measure μ, its potential $\mu V(e)$ is defined by

$$\mu V(e) = \int V(x, e) d\mu(x)$$

provided that μV is a positive Radon measure.

We shall say that a kernel V is *continuous* if it is a positive