97. On Axiom Systems of Propositional Calculi. I

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In a series of our papers, we are mainly concerned with axiom systems of propositional calculi containing many valued propositional calculi and modal logic etc. Our first paper is a preliminary note for our discussions and contains some elementary remarks. In our first some papers, we shall treat two valued propositional calculus.

In this series, we shall use the well known symbolisms formulated by J. Lukasiewicz (see [1] and [2]), inasmuch as Lukasiewicz symbols are quite helpful and useful for our discussions in axiom systems of propositional calculi.

The small Latin letters denote the propositional variables, and the Greek letters denote theses derived from some given axiom system of propositional calculus. The capital Latin letters N and C denote negation and implication respectively.

To prove theses of a system of two valued propositional calculus, we use two fundamental rules of inference.

1) The rule of *substitution*. For a thesis α in a system, the expression β obtained by a correct substitution in α is also a thesis of the system.

2) The rule of *detachment* (modus ponens). If $C\alpha\beta$ and α are theses in the system, then β is a thesis in it.

Of course, $C\alpha\beta$ denotes that α implies β.

In our discussion, we take up the well known system of axioms: $CCpqCCqrCpr$, $CCNppp$, and $CpCNpq$ formulated by J. Lukasiewicz [1] as fundamental one. In his Elements [1], J. Lukasiewicz has proved all axioms in Frege, Russell and Hilbert systems (see the table below) from his axioms using the rules of substitution and detachment. Further J. Lukasiewicz states in his Elements [1] that his first system of axioms are modified by $CCpqCCqrCpr$, $CpCNpq$, and $CCNpqCCqpp$. The proof of the equivalence has not given in his Elements [1]. A simple proof of the equivalence is given by Y. Arai, one of our colleagues, and will be published in the second note of our papers.

J. Lukasiewicz has also given further two systems of axioms, and B. Sobociński has stated two new systems of axioms. On the other hand, we have some single axiom systems by J. Lukasiewicz and A, Tarski [2] and C. Meredith etc. (see A. N. Prior [3]).