96. On Theorems of Korovkin. II

By Ritsuo NAKAMOTO and Masahiro NAKAMURA
Department of Mathematics, Osaka Gakugei University
(Comm. by Kinjirô KUNUGI, M.J.A., June 12, 1965)

1. P. P. Korovkin [2; Th. 3] established, among many others, the following theorem:

THEOREM 1. *Let L_n be a positive linear operator which maps the space $C[a, b]$ of all functions continuous on the closed interval $[a, b]$ into itself for every $n=1, 2, \cdots$. If*

(1) $$\lim_{n\to\infty} L_n f = f, \qquad \textit{uniformly},$$

is satisfied by $f(t)=1$, t and t^2, then (1) *is true for every $f \in C[a, b]$.*

Since several concrete operators on $C[a, b]$ are positive and linear, Korovkin's theorem plays fundamental role in his theory of approximation; for example, the Bernstein operator

$$B_n f(t) = \sum_{k=0}^{n} f\left(\frac{k}{n}\right)\binom{n}{k} t^k (1-t)^{n-k}$$

is linear and positive on $[0, 1]$ for every $n>0$.

One of the proofs of Theorem 1 due to Korovkin is based on the following theorem [2; Th. 1] on the convergence of positive linear functionals on $C[a, b]$:

THEOREM 2. *If a sequence $\{\varphi_n\}$ of positive linear functionals on $C[a, b]$ satisfies*

(2) $$\lim_{n\to\infty} \varphi_n(1) = 1,$$

and

$$\lim_{n\to\infty} \varphi_n(h) = 0,$$

where $h(t)=(t-c)^2$, $a \leqq c \leqq b$, then

$$\lim_{n\to\infty} \varphi_n(f) = f(c),$$

for all $f \in C[a, b]$.

2. A few years ago, Marie and Hisashi Choda proved in [1] an abstract version of Theorem 2. To introduce their theorem, some elementary notions on B^*-algebras are required, cf. [3].

A commutative Banach algebra A is called a *B^*-algebra* if A has an involution $x \to x^*$ which satisfies $\| xx^* \| = \| x \|^2$ for all $x \in A$. An element of A is called *positive*, symbolically $a \geqq 0$, if there is an element $b \in A$ such as $a=bb^*$. If a transformation L which maps A into a B^*-algebra B is called *positive* if $La \geqq 0$ for every $a \geqq 0$. A *character* of A is a homomorphism of A onto complex numbers. A character of A determines uniquely a maximal ideal of A.