

125. On Axiom Systems of Propositional Calculi. IV

By Kiyoshi ISÉKI

(Comm. by Kinjirô KUNUGI, M.J.A., Sept. 13, 1965)

Recently, in his book [5], E. Mendelson gave an axiom system for two valued propositional calculus. His axiom system is written by Lukasiewicz symbols as follows:

- 1 $CpCqp$,
- 2 $CCpCqrCCpqCpr$,
- 3 $CCNpNqCCNppq$.

E. Mendelson [5] proved some tautologies by using the rules of inference and a metatheorem known as Herbrand deduction theorem: If Γ is a set of theses and p, q are theses and $\Gamma, p \vdash q$, then $\Gamma \vdash p \supset q$ (see J. Herbrand [2] or A. A. Mullin [6]).

In this note, we shall use only rules of substitution and detachment, and prove some theses.

The first two axioms 1 and 2 are theses 18 and 35 in J. Lukasiewicz [4] respectively. The axiom 3 is also a thesis in Lukasiewicz (L_1)-system (see Y. Imai and Iséki [3]). It follows from 49: $CCNpNqCqp$ and 15: $CCNpqCCqpp$ in [4]. To prove it, we shall use the following two fundamental theses:

- a) $CCqrCCpqCpr$,
- b) $CCpCqrCqCpr$.

These tautologies are theses 22 and 21 in [4] respectively.

- b) $p/CNpq, q/Cqp, r/p *C15 p/q, q/p-4$,
- 4 $CCpqCCNqppq$.
- a) $p/CNqNp, q/Cpq, r/CCNqppq *C4-C49 p/q, q/p-3$,
- 3 $CCNqNpCCNqppq$.

This shows that axiom 2 is a thesis in L_1 -system. In the third note [1], Y. Arai has proved that $CpCqp, CCpCqrCCpqCpr$ imply the following important theses:

- 1' $CCpqCCqrCpr$,
- 2' $CCqrCCpqCpr$,

and

- 3' $CCpCqrCqCpr$.

We shall now proceed to prove the Lukasiewicz (L_1)-axioms 1: $CCpqCCqrCpr$, 2: $CCNppp$, and 3: $CpCNpq$.

From remarks given above, we have $CCpqCCqrCpr$. We shall show that axioms 1, 2, and 3 imply $CCNppp$. The proof is done by the following lines.