124. On Axiom Systems of Propositional Calculi. III

By Yoshinari ARAI

(Comm. by Kinjirô KUNUGI, M.J.A., Sept. 13, 1965)

In this note, we shall concern with Lukasiewicz (L_3) axioms (see Y. Imai and K. Iséki [1]). As mentioned in our previous papers, we only use the rules of substitution and detachment. The fundamental axioms are the following three theses:

1 $CpCqp$,

2 $CCpCqrCCpqCpr$,

3 $CCNpNqCqp$.

We shall first give a proof of $(L_3)\Rightarrow(L_1)$. From the (L_3)-system, we have the following theses:

1 $p/CCNqNpCpq$, q/Np $*C3$ p/q, q/p—4,

4 $CNpCCNqNpCpq$.

2 p/Np, $q/CNqNp$, r/Cpq $*C4$—$C1$ p/Np, q/Nq—5,

5 $CNpCpq$.

2 p/Nq, r/p $*C5$ p/q, q/p—6,

6 $CCNqqCNqp$.

1 $p/CCNqqCNqp$ $*C6$—7,

7 $CqCCNqqCNqp$.

2 p/q, $q/CNqq$, $r/CNqp$ $*C7$—$C1$ p/q, q/Nq—8,

8 $CqCNqp$.

8 q/p, p/q $*C9$,

9 $CpCNpq$.

1 $p/CCpCqrCCpqCpr$, q/Cqr $*C2$—10,

10 $CCqrCCpCqrCCpqCpr$.

2 p/Cqr, $q/CpCqr$, $r/CCpqCpr$ $*C10$—$C1$ p/Cqr, q/p—11,

11 $CCqrCCpqCpr$.

2 p/Cqr, q/Cpq, r/Cpr $*C11$—12,

12 $CCCqrCpqCCqrCpr$.

1 $p/CCCqrCpqCCqrCpr$, q/Cpq $*C12$—13,

13 $CCpqCCCqrCpqCCqrCpr$.

2 p/Cpq, $q/CCqrCpq$, $r/CCqrCpr$ $*C13$—$C1$ p/Cpq, q/Cqr—14,

14 $CCpqCCqrCpr$.

2 $p/CpCqr$, q/Cpq, r/Cpr $*C2$—15,

15 $CCCpCqrCpqCCpCqrCpr$.

2 r/p $*C1$—16,

16 $CCpqCpp$.