120. *On Infinitesimal Linear Isotropy Group of an Affinely Connected Manifold*

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(Comm. by Kinjirô KUNUGI, M.J.A., Sept. 13, 1965)

Introduction. Let M be a differentiable manifold with an affine connection of class C^∞. For each point p in M we denote by L_p the group of all linear transformations of the tangent space M_p at p. The *infinitesimal linear isotropy group* K_p is the subgroup of L_p consisting of all linear transformations of M_p which leave invariant the torsion tensor $(T)_p$, the curvature tensor $(R)_p$, and all their succesive covariant differentials $(\nabla T)_p$, $(\nabla^2 T)_p$, $\cdots$, $(\nabla R)_p$, $(\nabla^2 R)_p$, $\cdots$ [3]. Let $A(M)$ be the group of all affine automorphisms of M, H_p the subgroup of $A(M)$ consisting of all elements of $A(M)$ which fix the point p, and dH_p the linear isotropy group determined by H_p. In § 2, we shall investigate sufficient conditions that $dH_p = K_p$ at each p in M, and treat some applications. We discussed similar problems in a Riemannian manifold [6], [7]. Throughout this note we make use of the summation convention.

§ 1. Preliminaries. *Lemma* 1. *Let M be a differentiable manifold with an affine connection of class C^∞. If $f \in H_p$, then $(df)_p \in K_p$ at each p in M.*

Proof. Let B be the frame bundle of M, and let the structural equations be

$$d\tilde{\theta}^j = \tilde{\theta}^k \wedge \tilde{\theta}^j_k + \frac{1}{2} \tilde{P}^j_{km} \tilde{\theta}^k \wedge \tilde{\theta}^m,$$

$$d\tilde{\theta}^j_i = \tilde{\theta}^l_i \wedge \tilde{\theta}^j_l + \frac{1}{2} \tilde{S}^j_{ikm} \tilde{\theta}^k \wedge \tilde{\theta}^m.$$

f induces on B a transformation $\tilde{f}$ in the natural way. Taking a coordinate system $\{x^1, \cdots, x^n\}$ around p in M, we introduce a coordinate system $\{x^1, \cdots, x^n, X^1_1, \cdots, X^n_n\}$ in B. Then we have

$$\tilde{P}^j_{km} = \tilde{Y}^j_i \tilde{X}^p_k \tilde{X}^q_m T^i_{pq},$$

$$\tilde{P}^j_{km, m_t, \cdots, m_1} = \tilde{X}^{p_1}_{m_1} \cdots \tilde{X}^{p_t}_{m_t} \tilde{Y}^j_i \tilde{X}^p_k \tilde{X}^q_m \nabla_{p_1} \cdots \nabla_{p_t} T^i_{pq},$$

where the matrix $\|\tilde{Y}^i_j\|$ is the inverse matrix of $\|\tilde{X}^i_j\|$ and T^i_{pq} are the components of the torsion tensor T with respect to the coordinate system. Since f is an affine automorphism of M, we have

(1) $$\delta\tilde{f}\tilde{P}^j_{km} = \tilde{P}^j_{km}, \quad \delta\tilde{f}\tilde{P}^j_{km, m_t, \cdots, m_1} = \tilde{P}^j_{km, m_t, \cdots, m_1}.$$

Denoting by $\|a^i_j\|$ the matrix defined by $(df)_p(\partial/\partial x^j)_p = a^i_j(\partial/\partial x^i)_p$, and by $\|b^i_j\|$ the inverse matrix of $\|a^i_j\|$, we get from (1) that