119. *Singular Cut-off Process and Lorentz Properties*

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§ 1. **Introduction.** Usually it is believed that the ordinary cut-off disturbs the ordinary Lorentz covariance and causality condition defined in [4] pp. 249, 250. But by using a sort of singular cut-off which is alike to one defined in [3] p. 377, we can also avoid these difficulties in a sense. Namely, though these difficulties are too essential to avoid by the construction of $\sum_{i=1}^{n} C_i\varphi(x-x_i)$($\varphi(x)$; usual field function), the suitable use of E.R. ν integral for singular mollifier gives the positive effect for this purpose. This positive effect is one of the notable advantage of this E.R. ν singular cut-off. The direct purpose of the use of ν is the selection of the suitable conditional convergence for the definition of integral. It seems that the suitable interpretation of the change of ν used here is also possible from the consideration of inner structure of elementary particle. On the other hand, we can also avoid these difficulties by the change of the definition of Lorentz covariance and causality, which are likely to represent an approximation from another view point [2] p. 73. But in this article we do not treat this mainly. The materials of this article are arranged as follows: in § 2 E.R. ν integral is defined by A-integral form, in § 3 the definition of ν in the four (or three) dimensional E.R. ν singular cut-off and the positive effect of this singular cut-off to ordinary Lorentz covariance are shown, namely this positive effect is to rewrite the change of the smeared out field function by inhomogeneous Lorentz transform (inner product conserving) to the ordinary Lorentz covariant form (except the change of ν), and in § 4 the positive effect of E.R. ν singular cut-off to ordinary causality condition is given except the change of ν.

§ 2. **E.R. ν integral. Definition 1.** Suppose that the function $f(x)$ defined in the interval $[a, b]$ satisfies the following two conditions:

1) $\int_{[a,b]\cap[x;|f(x)|>n\phi(x)]} n\phi(x)dx$ tends to zero as n tends to ∞, where $\phi(x)$ is locally L^1 positive valued function,

2) there exists a finite limit $\lim_{n\to\infty}\int_a^b [f]_n(x)dx$, where

$$[f]_n(x)=\begin{cases} f(x) & \text{for } x \text{ satisfying the relation } |f(x)|\leqq n\phi(x) \\ 0 & \text{for } x \text{ satisfying the relation } |f(x)|>n\phi(x). \end{cases}$$

Then we say that $f(x)$ is E.R. ν integrable and that the above limit is E.R. ν integral of $f(x)$ (same as one by K. Kunugi etc.) which