115. *The Characters of Irreducible Representations of the Lorentz Group of n-th Order.*

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1. The connected component of the identity element of the orthogonal group associated with the indefinite quadratic form $x_1^2+x_2^2+\cdots+x_{n-1}^2-x_n^2$ is called the Lorentz group of n-th order and denoted by L_n.

We use the same definitions and notations as in [1] and [2]. In these papers we discussed infinite dimensional algebraically irreducible representations of the Lie algebra of L_n. We can prove that there exist complete irreducible representations of the group L_n in Hilbert spaces, which correspond to the representations of the Lie algebra listed in [2].

In this note we give the explicite formulae of the characters of these irreducible representations. Representations $\mathfrak{D}_{(\alpha;c)}$ can be constructed by the method of "induced representation" and in the series of thus constructed induced representations, some exceptional ones are not irreducible and they split into irreducible representations $\mathfrak{S}_\mu$, $\mathrm{D}^j_{(\alpha;p)}$, $\mathrm{D}^+_{(\alpha;p)}$, or $\mathrm{D}^-_{(\alpha;p)}$ (semi-reducible). The diagrams of these splitting are known from the infinitesimal stand point. The characters of the induced representations are calculated by integration of some integral kernels. Using the thus calculated characters of $\mathfrak{D}_{(\alpha;c)}$ and the character formulae of finite dimensional representations, we can obtain, for instance when $n=2k+3$, successively the characters of $\mathrm{D}^k_{(\alpha;p)}, \mathrm{D}^{k-1}_{(\alpha;p)}, \cdots, \mathrm{D}^1_{(\alpha;p)}$ and of the direct sum of $\mathrm{D}^+_{(\alpha;p)}$ and $\mathrm{D}^-_{(\alpha;p)}$. It needs some additional discussions to obtain the character of each $\mathrm{D}^+_{(\alpha;p)}$ and $\mathrm{D}^-_{(\alpha;p)}$ separately.

Here our discussions are restricted on one-valued representations, but the analogous results can be obtained for two-valued representations by the same method.

2. First we consider the case when n is odd: $n=2k+3$ $(k=0, 1, 2, \cdots)$.

The regular elements of L_n are divided into two classes G_1 and G_2. Every element $g\in G_1$ has eigenvalues $1, e^{i\varphi_1}, e^{-i\varphi_1}, \cdots, e^{i\varphi_k}, e^{-i\varphi_k}, e^t$, and e^{-t} (three of them are real positive) and we put $\lambda_r=e^{i\varphi_r}$, $\lambda_{-r}=e^{-i\varphi_r}$ $(r=1, 2, \cdots, k)$, $\lambda_{k+1}=e^t$, and $\lambda_{-(k+1)}=e^{-t}$. For $g\in G_2$, its eigenvalues are $1, e^{i\varphi_1}, e^{-i\varphi_1}, \cdots, e^{i\varphi_{k+1}}$, and $e^{-i\varphi_{k+1}}$ (all except 1 are complex)