

145. On Axiom Systems of Propositional Calculi. VII

By Yoshinari ARAI and Kiyoshi ISÉKI

(Comm. by Kinjirô KUNUGI, M.J.A., Oct. 12, 1965)

In his paper [1], Y. Arai, one of the present authors, obtained deductions of several axiom systems for propositional calculus from the (L_3) -axioms:

- 1 $CpCqp$,
- 2 $CCpCqrCCpqCpr$,
- 3 $CCNpNqCqp$.

In this note, we shall show the other deductions from the (L_3) -axioms and give some remarks. For the deductions, we use rules of substitution and detachment as in our previous notes.

The first two axioms imply the following theses:

- 1 $p/CCpCqrCCpqCpr, q/Cqr$ *C2—4,
- 4 $CCqrCCpCqrCCpqCpr.$
 - 2 $p/Cqr, q/CpCqr, r/CCpqCpr, *C4—C1$ $p/Cqr,$
 $q/p—5,$
- 5 $CCqrCCpqCpr.$
 - 1 $p/CpCpq, q/CCpqCpr$ *C1 $p/q, q/p—6,$
- 6 $CCCpqCprCqCpq.$
 - 5 $p/q, q/Cpq, r/Cpr—7,$
- 7 $CCCpqCprCCqCpqCqCpr.$
 - 2 $p/CCpqCpr, q/CqCpq, r/CqCpr$ *C7—C6—8,
- 8 $CCCpqCprCqCpr.$
 - 5 $p/CpCqr, q/CCpqCpr, r/CqCpr$ *C8—C2—9,
- 9 $CCpCqrCqCpr.$
 - 9 $p/Cqr, q/Cpq, r/Cpr$ *C5—10,
- 10 $CCpqCCqrCpr.$
 - 2 $q/p, r/q—11,$
- 11 $CCpCpqCCppCpq.$
 - 2 $q/Cqp, r/p$ *C1 $q/Cqp—C1—12,$
- 12 $Cpp.$
 - 9 $p/CpCpq, q/Cpp, r/Cpq$ *C11—C12—13,
- 13 $CCpCpqCpq.$

In the deductions from axioms 1 and 2, theses 5, 9, 10, and 13 are fundamental and important. To obtain thesis 10, we shall show another deduction by Y. Arai.

- 2 $p/Cqr, q/Cpq, r/Cpr$ *C5—14,
- 14 $CCCqrCpqCCqrCpr.$