144. On Axiom Systems of Propositional Calculi. VI

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In this note, we shall continue to deduce axiom systems quoted in [2], [3] from the Russell axiom system. Our considerations are needed in formulations of some algebraic models of propositional calculi. These formulations will be given in the future papers.

The Russell axiom system is as follows:

1   $CpCqp$,
2   $CCpqCCqrCpr$,
3   $CCpCqrCqCpr$,
4   $CNNpp$,
5   $CCpNpNp$,
6   $CCpNqCqNp$.

We shall first prove that the above axioms imply axiom systems by Frege, and Lukasiewicz (L_1) by using the rules of substitution and detachment.

        3 r/p *C1—7,
7   $CqCpp$.

        7 $q/CpCqp$ *C1—8,
8   Cpp.

        6 p/Nq *C8 p/Nq—9,
9   $CqCNNq$.

        3 p/Cpq, q/Cqr, r/Cpr *C2—10,
10   $CCqrCCpqCpr$.

        10 r/NNq *C9—11,
11   $CCpqCpNNq$.

        2 p/Cpq, $q/CpNNq$, $r/CNqNp$ *C11—C6 q/Nq—12,
12   $CCpqCNqNp$.

        2 p/Cpq, $q/CCqrCpr$, r/s *C2—13,
13   $CCCCqrCprsCCpqs$.

        13 $s/CCCprsCCqrs$ *C2 p/Cqr, q/Cpr, r/s—14,
14   $CCpqCCCprsCCqrs$.

        3 p/Cpq, $q/CCprs$, $r/CCqrs$ *C14—15,
15   $CCCprsCCpqCCqrs$.

        2 p/NNp, q/p, r/q *C4—16,
16   $CCpqCNNpq$.

        2 p/Cpq, $q/CNNpq$ *C16—17,
17   $CCCNNpqrCCpqr$.