179. On Branching Markov Processes

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In this paper we give a definition of branching Markov processes in terms of a property of the semi-group of Markov processes and describe several equivalent formulations. Our definition is a generalization of Kolmogoroff and Dmitriev's one [2] and would clarify the situation discussed in Skorohod [4] (cf. [3]). This equivalence plays an important role in studies of branching Markov processes.

1. Definition of branching Markov processes. Let S be a compact Hausdorff space satisfying the axiom of the second countability, therefore it is metrizable. We denote the metric of S by ρ_1. Let us denote by $S^{(n)}$ the n-fold product of S with product topology and the symmetrization of $S^{(n)}$ by S^n, i.e. S^n is the quotient space $S^{(n)}/R$ of $S^{(n)}$ by the equivalence relation R of permutation with quotient topology, therefore S^n is also metrizable. We denote the metric of S^n by ρ_n. Moreover, $S^0=\{\partial\}$, where ∂ is an extra point. This procedure is due to Moyal [3].

Let γ be the natural mapping from $\bigcup_{n=0}^{\infty} S^{(n)}$ to $\bigcup_{n=0}^{\infty} S^n$. We introduce a metric ρ in $\bigcup_{n=0}^{\infty} S^n$ defined by the formula

$$\rho(x, y)=\begin{cases}\dfrac{1}{2}\dfrac{\rho_n(x, y)}{1+\rho_n(x, y)}, & \text{if } x, y\in S^n, \\ |n-m|, & \text{if } x\in S^n,\ y\in S^m, \text{ and } n\neq m.\end{cases} \tag{1.1}$$

We denote the one-point compactification of $\bigcup_{n=0}^{\infty} S^n$ by $\boldsymbol{S}=\bigcup_{n=0}^{\infty} S^n\cup\{\Delta\}$.

In the following, $\boldsymbol{B}(S)$ (resp. $\boldsymbol{B}(\boldsymbol{S})$) is the space of bounded and Borel measurable functions on S (resp. $\boldsymbol{S}$) and $\boldsymbol{B}^*(S)$ is the subset of $\boldsymbol{B}(S)$ formed by functions f with $\|f\|\leqq 1$. We define a mapping $\wedge$ from $f\in\boldsymbol{B}^*(S)$ to $\hat{f}\in\boldsymbol{B}(\boldsymbol{S})$ by

$$\hat{f}(\boldsymbol{x})=\begin{cases}1, & \text{if } \boldsymbol{x}=\partial, \\ f(x_1)f(x_2)\cdots f(x_n), & \text{if } \boldsymbol{x}\in S^n \text{ and } \boldsymbol{x}\ni(x_1, x_2, \cdots, x_n), \\ 0, & \text{if } \boldsymbol{x}=\Delta.\end{cases} \tag{1.2}$$

Definition 1.1. A strong Markov process $X=\{x_t(w), \zeta, N_t, P_{\boldsymbol{x}}\}$[1)] on $\boldsymbol{S}$ is said to be a *branching Markov process* if its semi-group $\{T_t; t\geqq 0\}$ satisfies

1) Cf. e.g. [1]. We always assume that paths are right continuous and have left limits, and $\zeta=\infty$.