176. Algebraic Formulations of Propositional Calculi

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In this note, we shall concern with the Frege (F)-system and the Lukasiewicz (L_3)-system. As well known, the (L_3)-system:

1    $CpCqp$,
2    $CCpCqrCCpqCpr$,
3    $CCNpNqCqp$

characterizes two valued classical propositional calculus. In the (F)-system, the third axiom $CCNpNqCqp$ are replaced into three axioms: $CCpqCNqNp$, $CNNpp$, and $CpNNp$ and these five axioms give a complete axiom system for two valued propositional calculus.

If we take three axioms:

1    $CpCqp$,
2    $CCpCqrCCpqCpr$,
3' $CCpNqCqNp$,

we can deduce Cpp and $CCpqCNqNp$. As already shown in [1] and [2], from axioms 1 and 2, we have

4   Cpp,
5    $CCpqCCqrCpr$,

and

6    $CCqrCCpqCpr$.

Then we have the following theses:

       3' p/Nq *C4 p/q—7,
7    $CqNNq$.
       6 r/NNq *C7—8,
8    $CCpqCpNNq$.
       5 p/Cpq, $q/CpNNq$, $r/CNqNp$ *C8—C3' q/Nq—9,
9    $CCpqCNqNp$.

On the other hand, if we take

1    $CpCqp$,
2    $CCpCqrCCpqCpr$,
3'' $CCNpqCNqp$.

From the remark above, we have the theses 4, 5, and 6 by the axioms 1 and 2. Further we have the following theses by the same techniques above:

       3'' q/Np *C4—7,
7    $CNNpp$.
       5 p/NNp, q/p, r/q *C7—8,
8    $CCpqCNNpq$.