

175. On Axiom Systems of Propositional Calculi. X

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In the first note of our papers, we deduced (L_1) , (L_2) , (S_1) , and (S_2) -axioms from (H) -axioms. Further the authors of the first note gave a proof of $(H) \Rightarrow CCpCqrCCpqCpr$ (for notations, see [1]). In this note, we shall prove $(H) \Rightarrow (F)$, (R) , and (L_3) . In these systems, theses $CCpqCNqNp$, $CCpNqCqNp$, $CCNpqCNqp$, and $CCNpNqCqp$ are fundamental. An essential part of this note is to give proofs of these expressions.

The axioms of (H) -system are:

- 1 $CpCqp$,
- 2 $CCpCqrCqCpr$,
- 3 $CCqrCCpqCpr$,
- 4 $CpCNpq$,
- 5 $CCpqCCNpqq$.

Then we have the following theses applying the rules of substitution and detachment.

- 2 $p/Cpq, q/CNpq, r/q$ *C5—6,
- 6 $CCNpqCCpqq$.
- 2 r/p *C1—7,
- 7 $CqCpp$.
- 7 $q/CpCqp$ *C1—8,
- 8 Cpp .
- 6 q/Np *C8 p/Np —9,
- 9 $CCpNpNp$.
- 6 $q/CNNpq$ *C4 p/Np —10,
- 10 $CCpCANNpqCANNpq$.
- 10 q/p *C1 q/NNp —11,
- 11 $CANNpp$.
- 2 $p/Cqr, q/Cpq, r/Cpr$ *C3—12,
- 12 $CCpqCCqrCpr$.
- 12 $q/CNpq$ *C4—13,
- 13 $CCCNpqrCpr$.
- 13 $q/NNp, r/NNp$ *C9 p/Np —14,
- 14 $CpNNp$.
- 12 $p/Cpq, q/CCqrCpr, r/s$ *C12—15,
- 15 $CCCCqrCprsrCCpqs$.
- 15 $s/CCCprsrCCqrs$ *C12 $p/Cqr, q/Cpr, r/s$ —16,
- 16 $CCpqCCCprsrCCqrs$.