174. On Axiom Systems of Propositional Calculi. IX

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In this note, we shall prove that the Frege axioms 1∼5 of propositional calculus imply (H), (R), (L_1), (L_2), (L_3), (S_1), (S_2), and (M) axiom systems (for these notations and rules, see [2]). Y. Arai and K. Iséki, in their note [1], published that the (F)-system is equivalent to (L_3). They have shown that the (L_3)-system implies *CCNppp*. So, we do not prove that the (F)-system implies *CCNppp*, but we use this thesis.

1 *CpCqp*.
2 *CCpCqrCCpqCpr*.
3 *CCpqCNqNp*.
4 *CNNpp*.
5 *CpNNp*.
1 *p*/*CCpCqrCCpqCpr*, *q*/*Cqr* **C*2—6,
6 *CCqrCCpCqrCCpqCpr*.
2 *p*/*Cqr*, *q*/*CpCqr*, *r*/*CCpqCpr* **C*6—*C*1 *p*/*Cqr*, *q*/*p*—7,
7 *CCqrCCpqCpr*.
2 *p*/*Cqr*, *q*/*Cpq*, *r*/*Cpr* **C*7—8,
8 *CCCqrCpqCCqrCpr*.
1 *p*/*CCCqrCpqCCqrCpr*, *q*/*Cpq* **C*8—9,
9 *CCpqCCCqrCpqCCqrCpr*.
2 *p*/*Cpq*, *q*/*CCqrCpq*, *r*/*CCqrCpr* **C*9—*C*1 *p*/*Cpq*, *q*/*Cqr*—10,
10 *CCpqCCqrCpr*.
10 *q*/*NNp*, *r*/*q* **C*5—11,
11 *CCNNpqCpq*.
7 *r*/*NNq* **C*5 *p*/*q*—12,
12 *CCpqCpNNq*.
7 *q*/*NNq* **C*4—13,
13 *CCpNNqCpq*.
10 *p*/*CNqp*, *q*/*CNpNNq*, *r*/*CNpq* **C*3 *p*/*Nq*, *q*/*p*—*C*13 *p*/*Np*—14,
14 *CCNpqCNqp*.
10 *p*/*CNpNq*, *q*/*CNNqp*, *r*/*Cqp* **C*14 *q*/*Nq*—*C*11 *p*/*q*, *q*/*p*—15,
15 *CCNpNqCqp*.
1 *p*/*CCNqNpCpq*, *q*/*Np* **C*15 *p*/*q*, *q*/*p*—16,
16 *CNqCCNqNpCpq*.