201. Some Applications of the Functional-Representations of Normal Operators in Hilbert Spaces. XIX

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We next discuss the case where the ordinary part of $T(\lambda)$ is a polynomial of degree d.

Theorem 52. Let $T(\lambda)$ and σ be the same notations as before; let the ordinary part $R(\lambda)$ of $T(\lambda)$ be a polynomial in λ of degree d; let c be any finite complex number; let $n_d(\rho, c)$ denote the number of all the c-points, with due count of multiplicity, of $T(\lambda)$ in the domain $\Delta_\rho\{\lambda : \rho<|\lambda|<\infty\}$ with $\sigma<\rho<\infty$; let e_d denote the coefficient of λ^d in the expansion of $R(\lambda)$; let

$$N_d(\rho, c)=\int_\rho^\infty \frac{n_d(r, c)}{r} dr \ (\sigma<\rho<\infty);$$

let

$$m_d(\rho, c)=\frac{1}{2\pi}\int_0^{2\pi} \log \frac{1}{[T(\rho e^{-it}), c]} dt \ (\sigma<\rho<\infty);$$

and let

$$m_d(\infty, c)=\lim_{\rho\to\infty} m_d(\rho, c)(=\log\sqrt{1+|c|^2}).$$

Then the equality

$$N_d(\rho, c)+m_d(\rho, c)-m_d(\infty, c)+\log|e_d|=\frac{1}{2\pi}\int_0^{2\pi}\log\frac{\sqrt{1+|T(\rho e^{-it})|^2}}{\rho^d} dt$$

holds for every finite value c and every ρ with $\sigma<\rho<\infty$; and both the left and right sides of this equality converge to $\log|e_d|$ as ρ becomes infinite.

Proof. Suppose that $R(\lambda)=\sum_{\mu=0}^{d} e_\mu\lambda^\mu$, $(e_d\neq 0)$, and consider the function $g(\lambda)$ defined by

$$g(\lambda)=\begin{cases}\lambda^d\left[T\left(\frac{1}{\lambda}\right)-c\right] \left(0<|\lambda|\leqq\frac{1}{\rho}, \sigma<\rho<\infty\right)\\ e_d \ (\lambda=0).\end{cases}$$

Then $g(\lambda)=\sum_{\mu=0}^{d} e_\mu\lambda^{d-\mu}+\sum_{\mu=1}^{\infty} C_{-\mu}\lambda^{d+\mu}-c\lambda^d$ where $C_{-1}, C_{-2}, C_{-3}, \cdots$ are the coefficients stated at the beginning of the proof of Theorem 47, and $g(\lambda)$ is regular in the closed domain $\left\{\lambda : 0\leqq|\lambda|\leqq\frac{1}{\rho}\right\}$. If we now denote all the zeros, repeated according to the respective orders,