

## 198. On Axiom Systems of Propositional Calculi. XIII

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We know some single axioms of the classical propositional calculus. In this note, we shall show that Lukasiewicz-Tarski single axiom of the propositional calculus (see [2]) is equivalent to some axiom systems, for example,  $(L_3)$ -system. In their paper, J. Lukasiewicz and A. Tarski do not give the proof of equivalences. For notations and rules of inferences, see [3].

The fundamental Lukasiewicz-Tarski axiom is the following single thesis:

$$1 \quad CCCpCqpCCCNrCsNtCCrCsuCCtsCtuvCwv.$$

First, we shall prove that the Lukasiewicz-Tarski axiom implies  $(L_3)$ -system. The proof is not so easy. Therefore, for the proofs of theses 2 and 4, we shall write the results of substitutions.

- $$1 \quad p/CqCrq, q/CCNrCsNtCCrCsuCCtsCtu, v/CqCrq, \\ w/p *C1 \quad p/q, q/r, v/CqCrq, w/CCNrCsNtCCrCsu \\ CCtsCtu-2, \\ CCCCqCrqCCCNrCsNtCCrCsuCCtsCtuCqCrqCCCNr \\ CsNtCCrCsuCCtsCtuCqCrqCpCqCrq,$$
- $$2 \quad CpCqCrq. \\ 2 \quad p/CpCqCrq, q/p, r/q *C2-3,$$
- $$3 \quad CpCqp. \\ 1 \quad v/CCpCqpCCNrCsNtCCrCsuCCtsCtu, w/CpCqp *C2 \\ p/CpCqp, q/CCNrCsNtCCrCsuCCtsCtu, r/CpCqp-C3 \\ -C3-4, \\ CCCpCqpCCCNrCsNtCCrCsuCCtsCtuCCpCqpCCNr \\ CsNtCCrCsuCCtsCtuCCpCqpCCpCqpCCNrCsNtCCr \\ CsuCCtsCtu,$$
- $$4 \quad CCNrCsNtCCrCsuCCtsCtu. \\ 4 \quad r/p, s/q, t/p, u/r *C3 \quad p/Np-5,$$
- $$5 \quad CCpCqrCCpqCpr. \\ 5 \quad r/p, q/Cqp *C3 \quad q/Cqp-C3-6,$$
- $$6 \quad Cpp. \\ 3 \quad p/CCpCqrCCpqCpr, q/Cqr *C5-7,$$
- $$7 \quad CCqrCCpCqrCCpqCpr \\ 5 \quad p/Cqr, q/CpCqr, r/CCpqCpr *C7-C3 \quad p/Cqr, q/p-8,$$
- $$8 \quad CCqrCCpqCpr. \\ 5 \quad p/Cqr, q/Cpq, r/Cpr *C8-9,$$
- $$9 \quad CCCqrCpqCCqrCpr.$$