197. On Axiom Systems of Propositional Calculi. XII

By Yoshinari ARAI

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Our purpose in this paper is twofold: first, to prove that Luka-siewicz second axiom system of propositional calculus implies his first axioms, and second, to show that the axiom of (L_2)-system derives (F), (H), (L_3), (M), (R), (S_1), and (S_2) axiom systems. For the notations and rules of inference, see [1]. The fundamental axioms are the following three theses:

1    $CCCpqrCNpr$,
2    $CCCpqrCqr$,
3    $CCNprCCqrCCpqr$.

We shall first give a proof of $(L_2) \Rightarrow (L_1)$. From the (L_2)-system, we have the following theses:

$\qquad\qquad$ 2 p/Crq, q/p, r/Cqp *C2 p/r, r/p—4,

4    $CpCqp$.

$\qquad\qquad$ 1 q/p, $r/CsCpp$ *C4 p/Cpp, q/s—5,

5    $CNpCsCpp$.

$\qquad\qquad$ 3 q/Cpp, $r/CCpCqpCpp$ *C5 $s/CpCqp$—C4 p/Cpp,
$\qquad\qquad$ $q/CpCqp$—C4 q/p—C4—6,

6   Cpp.

$\qquad\qquad$ 1 r/Cpq *C6 p/Cpq—7,

7    $CNpCpq$.

$\qquad\qquad$ 3 r/Cpr *C7 q/r—8,

8    $CCqCprCCpqCpr$.

$\qquad\qquad$ 1 p/q, q/Cpr, $r/CCpqCpr$ *C8—9,

9    $CNqCCpqCpr$.

$\qquad\qquad$ 2 q/r, $r/CCpqCpr$ *C4 p/Cpr, q/Cpq—10,

10    $CrCCpqCpr$.

$\qquad\qquad$ 3 p/q, q/r, $r/CCpqCpr$ *C9—C10—11,

11    $CCqrCCpqCpr$.

$\qquad\qquad$ 2 p/r, q/p, $r/CqCrp$ *C4 p/Crp—12,

12    $CpCqCrp$.

Theses 7, 12 and the axiom 1 of (L_2) are the axioms of (S_1)-system.

Our first proof, say, proving to deduce (L_1)-system from (L_2) axioms, would be run:

$\qquad\qquad$ 8 p/q, q/p, r/Crp *C12—C13,

13    $CCqpCqCrp$.