196. *On Axiom Systems of Propositional Calculi. XI*

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(Comm. by Kinjirô KUNUGI, M.J.A., Dec. 13, 1965)

In this note we shall prove that the Sobociński (S_1) axiom system of propositional calculus implies (F), (H), (L_1), (L_2), (L_3), (M), (R), and (S_2) axiom systems. The notations and two rules of inference are mentioned in our previous notes published in this Proceedings. The (S_1)-system is read as follows:

1 $CNpCpq$,
2 $CpCqCrp$,
3 $CCNprCCqrCCpqr$.

Now we have the following theses from three axioms above.

3 p/q, q/p, r/Cqr *$C1$ p/q, q/r—4,
4 $CCpCqrCCqpCqr$.
4 r/Crp *$C2$—5,
5 $CCqpCqCrp$.
5p/Cpq, q/Np, r/s *$C1$—6,
6 $CNpCsCpq$.
3 q/Cqr, $r/CqCpr$ *$C6$ q/r, s/q—$C5$ p/r, r/p—7,
7 $CCpCqrCqCpr$.
7 p/Cqp, r/Crp *$C5$—8,
8 $CqCCqpCrp$.
3 $r/CCqrCpr$ *$C6$ q/r, s/Cqr—$C8$ p/r, r/p—9,
9 $CCpqCCqrCpr$.
7 p/Cpq, q/Cqr, r/Cpr *$C9$—10,
10 $CCqrCCpqCpr$.
3 q/Cqr, $r/CCpqCpr$ *$C6$ q/r, s/Cpq—$C10$—11,
11 $CCpCqrCCpqCpr$.
3 $r/CsCpq$ *$C6$—$C2$ p/q, q/s, r/p—12,
12 $CCpqCsCpq$.
4 $p/CqCpq$, q/Cpq, r/Cpq *$C4$ p/q, q/p, r/q—$C12$ s/q—13,
13 $CCpqCpq$.
4 p/Cqr *$C13$ p/q, q/r—14,
14 $CCqCqrCqr$.
14 q/p, r/Cqp *$C2$ q/p, r/q—15,
15 $CpCqp$.
14 q/p, r/p *$C15$ q/p—16,
16 Cpp.