192. A Duality Theorem for Locally Compact Groups. I

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1. Let G be a locally compact group.

The purpose of this paper is to prove the following propositions 1, 2, and 3, which may be considered as an analogue of the Tannaka's duality theorem in the case of compact groups or its homogeneous spaces, in a similar but somewhat narrow sense as [1], [2], and [3].

Let $R=\{R_g\}$, $L=\{L_g\}$ be right and left regular representations respectively, which are realized on the space $L^2(G)$ of all squar summable functions with respect to a right Haar measure μ on G.

If a unitary representation $D=\{U_g, \mathfrak{H}\}$ of G and a complete orthonormal system $\Phi=\{\varphi_\alpha\}_\alpha$ in $\mathfrak{H}$ are given, it is easy to show that the correspondense

$$v\otimes f\to\{\langle U_g v, \varphi_\alpha\rangle f(g)\}_\alpha \tag{1}$$

generates an isometric map A from $\mathfrak{H}\otimes L^2(G)$ onto the discrete direct sum of $L^2(G)$ with multiplicity equal to the dimension of $\mathfrak{H}$, which maps $U_g\otimes R_g$ to direct sum of R_g. I.e.,

Lemma 1. *$D\otimes R$ is unitary equivalent to ΣR_α by the map A, where each R_α is unitary equivalent to R.*

Especially we denote by $A(\Phi)$ the isometric map generated by (1) for the case of $R\otimes R\sim\Sigma R_\alpha$ with respect to the system Φ.

Now we formulate the main propositions.

Proposition 1. *Let T is a unitary operator on $L^2(G)$, satisfying the following conditions:*

$$TL_g=L_gT \quad \text{for any } g \text{ in } G. \tag{2}$$

If $A(\Phi)(f_1\otimes f_2)=\{h_\alpha\}_\alpha$, then

$$A(\Phi)(Tf_1\otimes Tf_2)=\{Th_\alpha\}_\alpha. \tag{3}$$

Then there exists the unique element g_0 in G such that

$$T=R_{g_0}.$$

Let Ω be the set of all equivalence classes of unitary representations of G, and $D=\{U_g^D, \mathfrak{H}^D\}$ be a representative of class $\hat{D}$ in Ω. Consider an operator field $\boldsymbol{T}=\{T(D)\}$ over Ω, where $T(D)$ is a unitary operator in $\mathfrak{H}^D$. Such a $\boldsymbol{T}$ is called *admissible* if the conditions (4) and (5) are satisfied by $\boldsymbol{T}$.

$$U_1(T(D_1)\oplus T(D_2))U_1^{-1}=T(D_3), \tag{4}$$

$$U_2(T(D_1)\otimes T(D_2))U_2^{-1}=T(D_4), \tag{5}$$

for arbitrary map U_1 (resp. U_2) which gives an unitary equivalence