190. *Doubly Extended Geometries by Non-Connection Methods*

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(Comm. by Zyoiti SUETUNA, M.J.A., Dec. 13, 1965)

The most important problem of geometry seems to be a generalization of the "Erlanger Programm" of Felix Klein (1872) to the case of differentiable manifolds. E. Cartan consacrated almost whole of his life to investigations along the line of Lie groups to this purpose and gave a few local connection geometries parallel to the classical geometries in the sense of the "Erlanger Programm" but without arriving at his own main goal. S. S. Chern [9] and C. Ehresmann [10, 11], A. Lichnerowicz [12] and T. Ōtsuki [13–16] have attempted to establish a *global* theory of connections leading to the cross sections of the principal fibre bundles introducing connections in them.

In a series of previous papers of the present author ([1–8], [18–25]), he has established *extended geometries* corresponding to the 22 branches shown in the system on p. 247 of [22] (to be referred to by $*$). In case of the extended affine geometry (and for other branches of geometry *Mutatis mutandis*), he has discoverd the II-*geodesic curves corresponding to* $\omega^l_\mu(x)$:

(1) $$d(\omega^l/dt)/dt\equiv\omega^l_\lambda(\ddot{x}^\lambda+\Lambda^\lambda_{\mu\nu}(x)\dot{x}^\mu\dot{x}^\nu)=0,\qquad(\omega^l=\omega^l_\mu(x)dx^\mu),$$

where (x^λ) are the local coordinates in a subset U_α of a differentiable manifold $M=\bigcup_\alpha U_\alpha$, $|\omega^l_\mu|\neq 0$ in M, $(\Omega^\lambda_l\omega^l_\mu=\delta^\lambda_\mu\Longleftrightarrow\Omega^\lambda_k\omega^h_\lambda=\delta^h_k;\ d\omega^l_\mu-\Lambda^\lambda_{\mu\nu}\omega^l_\lambda dx^\nu=0,\ \Lambda^\lambda_{\mu\nu}=\Omega^\lambda_l\partial_\nu\omega^l_\mu\equiv-\omega^l_\mu\partial_\nu\Omega^\lambda_l,\ (\lambda,\mu,\cdots;l,h,\cdots=1,2,\cdots,n))$;

(2) $$d\xi^l=a^l dt=\omega^l,\ \xi^l=a^l t+c^l,\quad(a^l=\text{const.},\ c^l=\text{const.})$$

and adopted the curve $\xi^l=a^l t+c^l$ as the ξ^l-axis. As the equation $\xi^l=a^l t+c^l$ tells us, the II-geodesic curves behave as for meet and join like straight lines. From (2), it follows that $dx^\lambda/dt=a^m\Omega^\lambda_m$ along the II-geodesic curve corresponding to $\omega^l_\mu(x)$. (ξ^l) were called the II-*geodesic parallel coordinates*. When ξ^l and $\bar{\xi}^l$ stands for x^λ and ξ^l respectively, we had come to consider

(3) $d\bar{\xi}^l=a^l_m(\xi)d\xi^m$, $(|a^l_m|\neq 0)$, (4) $\bar{\xi}^l=a^l_m(\xi)\xi^m+a^l_0$, $(a^l_0=\text{const.})$.

The conditions for the correspondence of $d^2\bar{\xi}^l/dt^2=0$ and $d^2\xi^l/dt^2=0$:

(5) $$da^l_m(\xi)d\xi^m=0,\quad da^l_m(\xi)\xi^m=0.$$

The totality of the transformations of the type (4) forms an *extended affine transformation group. All the extended geometries tabulated in* $*$ *are realized in the differentiable manifold* $M=\bigcup_\alpha U_\alpha$ *and belong to the "Erlanger Programm" of F. Klein, so that connections*