11. A Note on Riemann's Period Relations. II

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1. **Basic notations.** Let W be a Riemann surface of infinite genus and (F_n) its exhaustion in Noshiro's sense (Noshiro [11]), then there exists on W a canonical homology basis of A-type with respect to (F_n) such that $A_1, B_1, \cdots, A_{k(n)}, B_{k(n)}$ form a canonical homology basis of F_n (mod ∂F_n) and $A_i \times B_j = \delta_i^j$, $A_i \times A_j = B_i \times B_j = 0$ (Ahlfors [3]). We denote such a basis by C.H.B. $(F_n)_A$. Especially when (F_n) is a canonical exhaustion, there exists a *special* C.H.B. $(F_n)_A$ which satisfies the following condition: *the cycles A_k, B_k with $k > k(n)$ lie outside of F_n for all n (Ahlfors-Sario [4])*. We denote such a special basis by C.H.B. $(F_n)_B^q$ and call it *a canonical homology basis of B-type* with respect to the exhaustion $(F_n)^q$, where $(F_n)^q$ denotes a canonical exhaustion of W.

Definition 1. *Let Γ_1, Γ_2 be two subspaces in Γ_h. We will say that the generalized bilinear relations between Γ_1 and Γ_2 hold with respect to (F_n) and C.H.B. $(F_n)_A$ if we have for all $\omega_1 \in \Gamma_1$ and all $\omega_2 \in \Gamma_2$*

$$(\omega_1, \omega_2^*) = \lim_{n\to\infty} \sum_{k=1}^{k(n)} \left(\int_{A_k} \omega_1 \int_{B_k} \bar{\omega}_2 - \int_{A_k} \bar{\omega}_2 \int_{B_k} \omega_1 \right). \tag{1.1}$$

Analogously we will say that the special bilinear relations between Γ_1 and Γ_2 hold if we have (1.1) *for $\omega_1 \in \Gamma_1$ with only a finite number of non zero periods.*

2. **Special bilinear relation.** Let $\sigma(A_k)$, $\sigma(B_k)$ be reproducing differentials of class Γ_{h0} associated with cycles A_k, B_k respectively, and let $\tilde{\sigma}(A_k)$, $\tilde{\sigma}(B_k)$ be regular distinguished reproducing differentials of class $\Gamma_{h0} \cap \Gamma_{hse}^*$ associated with cycles A_k, B_k respectively (Ahlfors-Sario [4]). For $\omega_1 \in \Gamma_1$ with only a finite number of non zero periods we define $T\omega_1$ and $\tilde{T}\omega_1$ as follows:

$$T\omega_1 = \sum_{k=1}^{\infty} b_k \sigma(A_k) - a_k \sigma(B_k), \qquad \text{(a finite sum)} \tag{2.1}$$

$$\tilde{T}\omega_1 = \sum_{k=1}^{\infty} b_k \tilde{\sigma}(A_k) - a_k \tilde{\sigma}(B_k), \qquad \text{(a finite sum)} \tag{2.2}$$

where $(A_i, B_i) =$ C.H.B. $(F_n)_A$, $a_k = \int_{A_k} \omega_1$, $b_k = \int_{B_k} \omega_1$.

Theorem 1. *The maximal class of Γ_2 such that the special bilinear relations between $\Gamma_1 = \Gamma_{h0}$ and Γ_2 hold, is closure $(\Gamma_{h0} + \Gamma_{he})$.*

Proof. We put $max\, \Gamma_2 = \Gamma'$. From the assumption we have for arbitrary $\omega_1 \in \Gamma_1 = \Gamma_{h0}$ and arbitrary $\omega_2 \in \Gamma'$