10. Notes on Commutative Archimedean Semigroups. I

By Takayuki TAMURA

Department of Mathematics, University of California, Davis

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1. Introduction. A commutative semigroup S is archimedean if and only if for any ordered pair of elements, (a, b), of S there are an element c of S and a positive integer n (depending on (a, b)) such that $a^n = bc$. The author proved the following theorem in [5] (or p. 136, [1]).

Theorem 1. *If S is a commutative cancellative archimedean semigroup without idempotent, then S is isomorphic onto the semigroup of all pairs of non-negative integers and elements of an abelian group G,*

$$\{(n, \alpha);\ n \in N,\ \alpha \in G\},\quad N=\{0, 1, 2, \cdots\}$$

with a non-negative integer valued function $I: G \times G \to N$ where the multiplication is defined by

$$(n, \alpha)(m, \beta)=(n+m+I(\alpha, \beta), \alpha\beta)$$

and I satisfies

(1.1) $I(\alpha, \beta)=I(\beta, \alpha)$

(1.2) $I(\alpha, \beta)+I(\alpha\beta, \gamma)=I(\alpha, \beta\gamma)+I(\beta, \gamma)$

(1.3) *For any $\xi \in G$, there is a positive integer m depending on ξ such that $I(\xi^m, \xi)>0$.*

(1.4) *$I(\varepsilon, \varepsilon)=1$, ε being the identity of G.*

Further S is homomorphic onto G, $S=\bigcup_{\alpha \in G} S_\alpha$ where each congruence class S_α is a linearly ordered set with respect to the ordering $x<y$ defined by $x=a^n y$ for some positive integer n for a fixed element a of S.

No satisfactory construction theory has been established in the following cases: commutative archimedean semigroups with zero and commutative archimedean semigroups without idempotent in which cancellation is not assumed, except special cases (see [4], [5], [7]).

This paper and the continuation reports the theory of construction of the two cases just mentioned without proof. These results would complete the construction theory of commutative archimedean semigroups in all cases. The detailed paper will be published elsewhere [6].

2. Group decomposition. In § 2 through § 4 S is assumed to be a commutative archimedean semigroup either without idempotent or with zero.