9. *On Mappings in Uniform Spaces over a Topological Semifield*

By Tomoko KONDA

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1. Introduction. In their articles [1] and [2], M. Ya. Antonovskii, V. G. Boltyanskii, and T. A. Sarymsakov have developed to the theory of the topological semifield.

In this paper which is based on theories given at the [1] and [2] we shall prove a theorem similar to that of T. A. Brown and W. W. Comfort (see [3]) in uniform spaces over a topological semifield.

The notation used here very closely approximates that of [1], [2], and [3].

2.1. Proposition. *Let* (X, ρ, E) *be a metric space over a topological semifield* E. *For each neighborhood of zero* U *in* E, *let* $U^*=\{(x, y) \mid \rho(x, y) \in U\}$. *Then the family* $\mathfrak{U}^*$ *of all sets of the form* U^* *determines a basis for the uniform space over a topological semifield* E.

2.2. Definition. Let $\mathfrak{U}$ be a basis of neighborhoods of zero for a topological semifield E, then $\mathfrak{U}$ is said to be ample if, whenever $x \in U \in \mathfrak{U}$, there is a $W \in \mathfrak{U}$ for which $x \in W \subset \overline{W} \subset U$.

2.3. Definition. Let $\mathfrak{U}^*$ be a basis for the uniform space over the topological semifield E and let f be a function on X into X. Then

(a) f is said to be a contraction with respect to $\mathfrak{U}^*$ if $(fx, fy) \in U^*$ whenever $(x, y) \in U^* \in \mathfrak{U}^*$;

(b) f is said to be an expansion with respect to $\mathfrak{U}^*$ if $(x, y) \in U^*$ whenever $(fx, fy) \in U^* \in \mathfrak{U}^*$;

(c) f is said to be isobasic with respect to $\mathfrak{U}^*$ if f is both a contraction with respect to $\mathfrak{U}^*$ and an expansion with respect to $\mathfrak{U}^*$.

3.1. Theorem. *Let* $\mathfrak{U}$ *be an open basis of neighborhoods of zero for the totally bounded Hausdorff metric space* (X, ρ, E) *and* $\mathfrak{U}^*$ *is a basis for the uniform space over the topological semifield* E. *If a function* f, *mapping* X *onto* X, *is a contraction with respect to* $\mathfrak{U}^*$ *and* $\mathfrak{U}$ *is ample then* f *is isobasic with respect to* $\mathfrak{U}^*$.

Proof. Let $(fx, fy) \in U^*$, since $\mathfrak{U}$ is ample, we find $W \in \mathfrak{U}$ such that $\rho(fx, fy) \in W \subset \overline{W} \subset U$. Suppose now that $(x, y) \notin U^*$. Then there is a symmetric $V_1^* \in \mathfrak{U}^*$ for which $(x, y) \notin V_1^* \circ W^* \circ V_1^*$. In fact, $\rho(x, y) \notin U$ and $\rho(x, y) \in E \setminus U \subset E \setminus \overline{W}$, the set $E \setminus \overline{W}$ is open and there is a symmetric neighborhood of zero V_1 such that $\rho(x, y) + V_1 + V_1 \subset E \setminus \overline{W}$. Since $E \setminus \overline{W} \subset E \setminus W$, we have $\rho(x, y) + V_1 + V_1 \subset E \setminus W$, i.e,