4. *Connection of Topological Fibre Bundles*

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In Asada [2], we give a general theory of connections of topological vector bundles. There a connection form $\{\theta_U\}$ of the given bundle ξ has following property: The value of $1+\theta_U$ belongs in G, the structure group of ξ. Therefore starting from $\{s_U\}=\{1+\theta_U\}$, we can construct a theory of connections of arbitrary topological fibre bundles without using the ring A of Asada [2]. To state this theory is the purpose of this note. But we don't know whether there exists or not a connection form for an arbitrary fibre bundle ξ.

1. *Connection of fibre bundles.* We denote by X a topological space, ξ a topological fibre bundle over X with structure group G. The transition functions of ξ are denoted by g_{UV}.

As in Asada [2], $n°1$, we denote the group of continuous maps from $V(\Delta_s(U))$ to G with equivalence relation $f_1\sim f_2$ if and only if $f_1| W=f_2|W$ for some neighborhood $W(\Delta_s(U))$ of $\Delta_s(U)$ in $U\times\cdots\times U$ by $\tilde{C}^s(U, G)$ and set

$C^s(U, G)=\{f|f\in\tilde{C}^s(U, G), f(\cdots, x_i, x_i, \cdots)=1$ for all i, $0\le i\le s-1\}$.

Then we define the sheaves $\tilde{\mathcal{G}}^r=\tilde{\mathcal{G}}^r(\xi)$ and $\mathcal{G}^r=\mathcal{G}^r(\xi)$ by

$\tilde{\mathcal{G}}^r$: the sheaf of germs of those maps $\{f_U\}, f_U\in\tilde{C}^r(U, G)$, $g_{UV}(x_0)^{-1}f_U(x_0, \cdots, x_r)g_{UV}(x_r)=f_V(x_0, \cdots, x_r)$.

$\mathcal{G}^r$: the subsheaf of $\tilde{\mathcal{G}}^r$ consisted those elements $\{f_U\}$ that $f_U\in C^r(U, G)$ for all U.

Definition. If $\{s_U\}\in H^0(X, \mathcal{G}^1)$, then we call $\{s_U\}$ is a connection form of ξ.

Note. As usual, if $\{s_U\}$ is a connection form of $\{g_{UV}\}$, $\{U'\}$ is a refinement of $\{U\}$ and $g_{U'V'}=g_{UV}\mid U'\cap V'$, then $\{s_{U'}\}, s_{U'}=s_U\mid U'$, becomes a connection form of $\{g_{U'V'}\}$. We identify $\{s_U\}$ and this $\{s_{U'}\}$. On the other hand, if $\{s_U\}$ is a connection form of $\{g_{UV}\}$ then $\{h_U(x_0)s_U(x_0, x_1)h_U(x_1)^{-1}\}$ is a connection form of $\{h_Ug_{UV}h_V^{-1}\}$. We identify $\{s_U\}$ and this $\{h_Us_Uh_U^{-1}\}$. For the simplicity, we identify $\{s_U\}$ and the equivalence class of $\{s_U\}$.

Lemma 1. *$H^0(X, \mathcal{G}^1)$ is non-empty if and only if $H^0(X, \tilde{\mathcal{G}}^1)$ is non-empty.*

Lemma 2. *$\{1\}$ belongs in $H^0(X, \mathcal{G}^1)$ if and only if $\{1\}$ belongs in $H^0(X, \tilde{\mathcal{G}}^1)$.*

Theorem 1. *ξ is equivalent to a bundle with tatally disconnect structure group if and only if $\{1\}$ becomes a connection form of ξ.*

Proof. If $\{h_Ug_{UV}h_V^{-1}\}$ is locally constant, then $\{s_U(x_0, x_1)\}=$