# 33. *Note on the Structure of Regular Semigroups*

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(Comm. by Kinjirô KUNUGI, M.J.A., Feb. 12, 1966)

§1. **Introduction.** Let $X=\{x_1, x_2, \cdots, x_n\}$ be a set in which each element $x_i$ is called a variable. By a *permutation identity* in the variables $x_1, x_2, \cdots, x_n$, we shall mean a form

(P) $$x_1x_2\cdots x_n = x_{p_1}x_{p_2}\cdots x_{p_n}$$

where $(p_1, p_2, \cdots, p_n)$ is a non-trivial permutation of $(1, 2, \cdots, n)$.[1] For example, *commutativity* $x_1x_2=x_2x_1$, *left* [*right*] *normality* $x_1x_2x_3=x_1x_3x_2$ [$x_1x_2x_3=x_2x_1x_3$] and *normality* $x_1x_2x_3x_4=x_1x_3x_2x_4$ are all permutation identities. If a subset $M$ of a semigroup $G$ satisfies the following condition (C.P), then we shall say that $M$ *satisfies* the permutation identity (P) in $G$:

(C.P) For any mapping $\varphi$ of $X$ into $M$, the equality

$$\varphi(x_1)\varphi(x_2)\cdots\varphi(x_n)=\varphi(x_{p_1})\varphi(x_{p_2})\cdots\varphi(x_{p_n})$$

is satisfied in $G$.

In particular if $G$ satisfies the permutation identity (P) in $G$, we simply say that $G$ satisfies the permutation identity (P). For example, a regular semigroup in which the set of idempotents satisfies commutativity is an inverse semigroup firstly introduced by Vagner [5] under the term 'generalized group' (see also [1], p. 28), and the structure of inverse semigroups was clarified by Preston [3] and [4]. A band (i.e., idempotent semigroup) satisfying [left, right] normality is called a [left, right] normal band, and the structure of [left, right] normal bands was also determined by Kimura and the author [6]. Each of an inverse semigroup and a [left, right] normal band is of course a regular semigroup in which the set of idempotents satisfies a permutation identity. The main purpose of this paper is to present a structure theorem for regular semigroups in each of which the set of idempotents satisfies a permutation identity. The complete proofs are omitted and will be given in detail elsewhere. Any symbol and terminology should be refered to [1] and [6], unless otherwise stated.

§2. **Generalized inverse semigroups.** Let $S$ be a regular semigroup. Then for each element $a$ of $S$, there exists an element $a^*$ such that $aa^*a=a$ and $a^*aa^*=a^*$. Such an element $a^*$ is called

1) The form (P) can be considered as the pair $(x_1x_2\cdots x_n, x_{p_1}x_{p_2}\cdots x_{p_n})$ of the two words $x_1x_2\cdots x_n$ and $x_{p_1}x_{p_2}\cdots x_{p_n}$.