27. *On Variants of Axiom Systems of Propositional Calculus. I*

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In this note, we shall show that any axiom system containing the *BCK*-system of propositional calculus may be effectively changed into a new system which has axioms less than the number of the original axioms. Following certain 'combinatory logicians', we put *B* for *CCqrCCpqCpr*, *C* for *CCpCqrCqCpr*, and *K* for *CpCqp*. This system was given by C. A. Meredith. And Prof. K. Iséki has given the algebraic formulation of the *BCK*-system (see, [5]). For the notations and two rules of inferences, see [4].

Theorem 1. *If F is a thesis in the BCK-system, then CCCpCqpCFvCwv having no occurrences of p, q, v, and w in F implies CpCqp and F.*

The result is obtained by the following proof line.

1 *CCCpCqpCFvCwv.*

1 *p/CpCqp, q/F, v/CpCqp* **C*1 *v/CpCqp, w/F*—2,

2 *CwCpCqp.*

2 *w/CCCpCqpCFvCwv* **C*1—3,

3 *CpCqp.*

1 *v/CCpCqpF, w/CpCqp* **C*2 *p/F, w/CpCqp*—*C*3—*C*3—4,

4 *F.*

Hence thesis 1 implies *CpCqp* and *F*, which completes the proof of Theorem 1.

Theorem 2. *If F is a formula in the BCK-system, then this system implies CCCpCqpCFvCwv, where p, q, v, and w do not contain in F.*

Proof. The axioms of the *BCK*-system are given by the following:

1′ *CCqrCCpqCpr*,

2′ *CCpCqrCqCpr*,

3′ *CpCqp.*

It is well known that these axioms imply (see, [1]),

4′ *CCpqCCqrCpr*,

5′ *CPCCpqq.*

Then we have the following theses:

5′ *p/F, q/v* **CF*—1,

1 *CCFvv*,