23. *Two-Dimensional Diffraction of Acoustic and Electromagnetic Waves by an Open Boundary*

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1. It is well known that the analyses of two-dimensional acoustic and electromagnetic fields are reduced to that of the Helmholtz equation

$$\Delta u + k^2 u = 0, \tag{1}$$

where u is a velocity potential in the case of acoustics and is a z-component of electric or magnetic fields in the case of electromagnetism. $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2$, k^2 ($Im.\ k \leqq 0$) is a constant, and a factor $e^{\gamma z + i\omega t}$ is supressed throughout. The boundary conditions

$$u = 0, \tag{2}$$

or

$$\frac{\partial u}{\partial n} = 0 \tag{3}$$

is prescribed on a given boundary L, where n is a given unit normal on L. Further, u is required to satisfy the radiation condition at infinity.

When L is a closed contour, the above mentioned problems have long been a subject of many investigations, and (1) has been solved by various techniques. In fact, when L is of particular geometry, say a whole circle or a whole straight line, (1) has been solved by means of a Fourier-series or a Fourier transform technique, and when L is of semi-infinite extent, it has been solved by a Wiener-Hopf technique [1]. When L is a closed contour of a general geometry, then (1) has been converted into a second kind integral equation of Fredholm over L, which may be solved by a conventional way.

However, when L is open, that is, when L is an open arc or a union of open arcs, say a circle or a line with arbitrary slots, no rigorous and general analysis has been studied on (1), and the problems have usually been solved approximately assuming that there is only one narrow slit in the boundary and that the distribution of field components in the slit is known [2], [3]. Since we find a lot of counterparts of the problems in practical fields, for example, in a theory of a slotted antenna and a leaky waveguide, a mathematical theory of the problems is worth studying, while the theory