21. *Regularity of Orbits Space on Semisimple Lie Groups*

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1. Let G be a semisimple Lie group, and KHN be its Iwasawa decomposition, M be the subgroup $K\cap\mathfrak{C}(H)$ where $\mathfrak{C}(H)$ shows the centralizer of H.

F. Bruhat [1] shows that $\Gamma=MHN$ is a closed subgroup of G, and G is a disjoint sum of finite Γ-Γ double cosets which correspond to elements of Weyl group in one-to-one way.

While denote by $G^t=G\times\cdots\times G$ the direct product of G with multiplicity t and by $\widetilde{G}_t=\{(g,\cdots,g)\in G^t\}$ the diagonal subgroup of G^t, which is isomorphic to G.

There exists a question whether Γ^t and $\widetilde{G}_t$ are regularly related in G^t or not, in the sense of Mackey [2]. This problem is related to a problem of decomposability of Kronecker product of induced representations of G by representations of Γ, with multiplicity t (cf. [3]).

The purpose of this work is to solve this problem affirmatively.

Proposition. *Γ^t and $\widetilde{G}_t$ are regularly related in G^t.*

2. Proof of the proposition. At first, we can equate $\Gamma^t\backslash G^t/\widetilde{G}_t$ to $\Gamma^{t-1}\backslash G^{t-1}/\widetilde{\Gamma}_{t-1}$ by the map of representatives of cosets, $G^t\ni(g_1,g_2,\cdots,g_t)\to(g_1g_t^{-1},g_2g_t^{-1},\cdots,g_{t-1}g_t^{-1})\in G^{t-1}$.

Using Glimms results [4], we can conclude that $\Gamma\backslash G/\Gamma$ is T_0 and the union of all lower dimensional Γ-Γ double cosets in G becomes a null set F in G, and $G'=G-F$ is open as a union of open cosets. Therefore it is sufficient to show the space $\Gamma^{t-1}\backslash(G')^{t-1}/\widetilde{\Gamma}_{t-1}$ is countably separated.

Again by [4], the last space is countably separated if and only if it is T_0. And for fixed l and closed subgroups $A\supset B$ in Γ^l, if $\Gamma^l\backslash(G')^l/A$ and $\hat{g}\Gamma^l\hat{g}^{-1}\cap A\backslash A/B$ are T_0 for any $\hat{g}$ in $(G')^l$, then $\Gamma^l\backslash(G')^l/B$ is T_0.

In this case, we put $A=\widetilde{\Gamma}_{l-1}\times\Gamma=\{(\gamma,\cdots,\gamma,\gamma')\in\Gamma^l\}$ and $B=\widetilde{\Gamma}_l$. Then easily we get, $\Gamma^l\backslash(G')^l/\widetilde{\Gamma}_{l-1}\times\Gamma\sim\Gamma^{l-1}\backslash(G')^{l-1}/\widetilde{\Gamma}_{l-1}\times\Gamma\backslash G'/\Gamma$ and $\hat{g}\Gamma^l\hat{g}^{-1}\cap A\backslash A/B\sim\Gamma^{l-1}(\hat{g})\times\Gamma^1(g_l)\backslash\Gamma\times\Gamma/\widetilde{\Gamma}_2\sim\Gamma^{l-1}(\hat{g})\backslash\Gamma/\Gamma^1(g_l)$, where $\Gamma^{l-1}(\hat{g})=\Gamma\cap g_1\Gamma g_1^{-1}\cap g_2\Gamma g_2^{-1}\cap\cdots\cap g_{l-1}\Gamma_{g_{l-1}}^{-1}$, and $\Gamma^1(g_l)=g_l\Gamma g_l^{-1}\cap\Gamma$, for $\hat{g}=(g_1,g_2,\cdots,g_l)$ in $(G')^l$. Consequently, if we prove $\Gamma^{l-1}(\hat{g})\backslash\Gamma/\Gamma^1(g_l)$ is T_0, then by the induction with respect to l, we get the proof.

Now we shall show that $\Gamma^1(g)$ is conjugate to MH in Γ for any