17. *Algebraic Proof of the Separation Theorem on Classical Propositional Calculus*

By Tsutomu HOSOI
Mathematical Institute, University of Tokyo
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In our previous paper [2], it is shown that we can so axiomatize the classical predicate calculus that the separation theorem, mentioned below, holds on it:

Separation Theorem: *A classically provable formula can be proved by using at most the axioms for implication and those of the connectives which actually appear in the formula.*

An example of the propositional fragment of such axiom systems is:

1.1 $p\supset q\supset p$.
1.2 $(p\supset q\supset r)\supset(p\supset q)\supset(p\supset r)$.
1.3 $((p\supset q)\supset p)\supset p$.
1.4 $p\&q\supset p$ and $p\&q\supset q$.
1.5 $(p\supset q)\supset(p\supset r)\supset(p\supset q\&r)$.
1.6 $p\supset p\vee q$ and $q\supset p\vee q$.
1.7 $(p\supset r)\supset(q\supset r)\supset(p\vee q\supset r)$.
1.8 $(p\supset\sim q)\supset(q\supset\sim p)$.
1.9 $\sim p\supset p\supset q$.

The rules of inference are modus ponens and the rule of substitution for variables. We associate to the right and assume the convention that implication binds less strongly than other connectives. This system is classical, for we obtain from 1.3 the law of excluded middle by putting $p=r\vee\sim r$ and $q=r\&\sim r$.

In [2], we proved the separation theorem by using Gentzen's cut elimination theorem on the classical predicate calculus. And in this paper is given an algebraic proof of the theorem on the propositional calculus defined above. The algebraic proof of the theorem on the intuitionistic propositional calculus was given by Horn [1]. And his system is obtained from our system by deleting 1.3. He defined algebras called I, IN, ID, IC, IDN, ICN, ICD, or ICDN and reduced the proof of the theorem to the problem of embedding each algebra into an ICDN algebra. We will not give the details of Horn's paper but borrow some definitions and results from it, so see his paper for those.

Horn first embedded algebras without N into those with N, which was not so complicated. And then he treated the algebras without C, and finally the case of algebras without D, which was the most