55. *Skewness in Special Semi-Modular Lattices*

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In his lattice-theoretic formulation of incidence geometry, Gorn [2] uses a Condition E, to be called skewness [3] here, without considering its independence from the other axioms. This note is a partial answer to this question. Lattice-theoretic notions not specifically defined will be in agreement with those of Birkhoff [1].

Gorn's incidence geometry is a relatively complemented semi-modular lattice of length greater than or equal to 5 and with a greatest lower bound 0. Let L with order $\leqq$ and operations $+, \cdot$ be such a lattice. The incidence geometry satisfies the further condition that it is *special* [3], i.e., if $a, b \in L$ such that $ab>0$, then $(a, b)M$ ((a, b) is a modular pair). Let L be special. L satisfies *skewness* means that if $l_1, l_2, l_3, l_4 \in L$ are lines (elements of L covering points) such that l_i+l_j covers l_i and l_j for $(i, j)=(1, 2), (1, 3), (2, 3), (2, 4), (3, 4)$, then l_1+l_4 covers l_1 and l_4.

Theorem. *If L is of length greater than* 5, *then L satisfies skewness.*

Proof. Let $l_1, l_2, l_3, l_4 \in L$ be lines such that l_i+l_j covers l_i and l_j for $(i, j)=(1, 2), (1, 3), (2, 3), (2, 4), (3, 4)$. Suppose l_1+l_4 does not cover l_1 or l_4, i.e., l_1 and l_4 are skew lines (see [2], p. 164). Define $h=l_1+l_4$. Then the closed interval from 0 to h is of length 5 and h covers l_i+l_j for $(i, j)=(1, 2), (1, 3), (2, 3), (2, 4), (3, 4)$. Let p be a point such that $p \nleqq h$, which exists since L is relatively complemented and of length greater than 5. Then $p+h$ covers h. Define $m_i=p+l_i$ for each i. Then the m_i are distinct, m_i covers l_i for each i, m_2+m_3 covers m_2 and m_3, and $(m_1, m_4)M$ and $(m_2, m_3)M$ since $m_1m_4, m_2m_3 \geqq p>0$. Further, $m_1+m_4=(p+l_1)+(p+l_4)=p+(l_1+l_4)=p+h$. If $m_2m_3>p$, then $m_1 \cdot m_4=(m_1+m_2)(m_1+m_3)\cdot(m_2+m_4)(m_3+m_4)=(m_1+m_2)(m_2+m_4)\cdot(m_1+m_3)(m_3+m_4)=m_2\cdot m_3>p$. Thus m_1, m_4 cover m_1m_4, but $m_1+m_4=p+h$ does not cover m_1, m_4 contrary to $(m_1, m_4)M$. Hence $m_2m_3=p$. Now m_2, m_3 do not cover m_2m_3 while m_2+m_3 covers m_2 and m_3. This is contrary to $(m_2, m_3)M$. Thus, necessarily, l_1+l_4 covers l_1 and l_4.