52. *On Axiom Systems of Propositional Calculi. XVI*

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(Comm. by Kinjirô KUNUGI, M.J.A., March 12, 1966)

In this paper, we shall show that several axiom systems of positive implicational calculus are equivalent. First we shall prove that 2-Axiom Base given by J. Lukasiewicz implies other axiom basis, i.e. 4-Axiom Base and 3-Axiom Base given by D. Hilbert, 2-Axiom Base and some 1-Axiom Basis given by C. A. Meredith (for example see, [2]).

For the details of the notations and the two rules of inferences for deductions, see [1].

The 2-Axiom Base given by J. Lukasiewicz is the set of the following two formulas.

1 $CpCqp$.

2 $CCpCqrCCpqCpr$.

Under these axioms, we have:

1 $p/CCpCqrCCpqCpr$, q/Cqr $*C2$—3,

3 $CCqrCCpCqrCCpqCpr$.

2 p/Cqr, $q/CpCqr$, $r/CCpqCpr$ $*C3$—$C1$ p/Cqr, q/p—4,

4 $CCqrCCpqCpr$.

2 p/Cqr, q/Cpq, r/Cpr $*C4$—5,

5 $CCCqrCpqCCqrCpr$.

1 $p/CCCqrCpqCCqrCpr$, q/Cpq $*C5$—6,

6 $CCpqCCCqrCpqCCqrCpr$.

2 p/Cpq, $q/CCqrCpq$, $r/CCqrCpr$ $*C6$—$C1$ p/Cpq, q/Cqr—7,

7 $CCpqCCqrCpr$.

1 $p/CqCpq$, $q/CCpqCpr$ $*C3$ p/q, q/p—8,

8 $CCCpqCprCqCpq$.

2 $p/CCpqCpr$, $q/CqCpq$, $r/CqCpr$ $*C4$ p/q, q/Cpq, r/Cpr—$C8$—9,

9 $CCCpqCprCqCpr$.

4 $p/CpCqr$, $q/CCpqCpr$, $r/CqCpr$ $*C9$—$C2$—10,

10 $CCpCqrCqCpr$.

2 $p/CpCqr$, q/Cpq, r/Cpr $*C2$—11,

11 $CCCpCqrCpqCCpCqrCpr$.

2 r/p $*C1$—12,

12 $CCpqCpp$.

12 q/Cqp $*C1$—13,