47. On the Existence of Competitive Equilibrium

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(Comm. by Kinjirô KUNUGI, M.J.A., March 12, 1966)

The purpose of this note is to show the existence of competitive equilibrium for an economy, where the excess demand function is supposed to be a point-to-set mapping, without the aid of fixed point theorems.[1]

First, the economic model in question will be specified with the help of the following notations and terminology, where all commodities are labeled $i=1, 2, \cdots, n$;

- X: the commodity space (mathematically, an n-dimensional Euclidean space R^n);[2]
- P: the set of price vectors (mathematically, a R^n_+ with the origin 0 deleted);
- $E(p)$: the excess demand function (mathematically, a point-to-set mapping from P into X).

$p^* \in P$ will be called an *equilibrium price vector*, if there exists $x^* \in E(p^*)$ such that $0 \geqq x^*$. Our main concern is with the existence of such equilibrium price vectors. To this end, the following assumptions may be imposed on $E(p)$:

(C) $E(p)$ is continuous on P, i.e., both upper semi-continuous and lower semi-continuous on P. Furthermore the set $E(p)$ is compact for all $p \in S$;

(H) $E(p)$ is positive homogeneous of degree zero, i.e.,

$$E(\lambda p)=E(p) \qquad \text{for all } \lambda>0 \text{ and } p \in P;$$

(W) The generalized Walras law holds, i.e.,

$$(p, x)^{3)} \leqq 0 \qquad \text{for all } p \in P \text{ and } x \in E(p);$$

(S) Weak gross substitutability prevails, i.e., $p \geqq q$ and $p_i=q_i$ imply that $x_i \geqq y_i$ holds for any $x \in E(p)$ and any $y \in E(q)$ $(i=1, 2, \cdots, n)$.

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1) Similar developments are found in the following papers. H. Nikaido: Generalized gross substitutability and extremization, in Advances in Game Theory, Princeton U. P., 55-68 (1964). K. Kuga: Weak gross substitutability and the existence of competitive equilibrium, in Econometrica, **33**, 593-599 (1965).

2) The element of R^n may be considered as the row vector. $0=(0, 0, \cdots, 0)$. $e=(1, 1, \cdots, 1)$. For $x=(x_1, x_2, \cdots, x_n)$ and $y=(y_1, y_2, \cdots, y_n)$ $x \geqq y$ means $x_i \geqq y_i$ for $i=1, 2, \cdots, n$. R^n_+ denotes the set $\{p \mid p \in R^n, p \geqq 0\}$. S denotes the set $\left\{p \mid p \in P, \sum_{i=1}^n p_i=1\right\}$.

3) $(p, x)=\sum_{i=1}^n p_i x_i$, where $p=(p_1, p_2, \cdots, p_n)$ and $x=(x_1, x_2, \cdots, x_n)$.