86. *On Locally Cyclic Semigroups*

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A locally cyclic semigroup is defined as follows:

Definition 1. A semigroup S is called locally cyclic if for every $a, b \in S$ there is an element $c \in S$ and positive integers m and n such that $a=c^m$ and $b=c^n$.

A locally cyclic semigroup is a commutative archimedean semigroup [1].

In Dean and Oehmke's paper [2], they proved that the lattice of congruences of a locally cyclic semigroup is a distributive lattice. However the structure of those semigroups has not been studied. In this note we will report the results of the study of the structure of locally cyclic semigroups without detailed proof.

A locally cyclic group is defined to be a locally cyclic semigroup which is a group. This is different from that given by [4]:

(1) A group G is called locally cyclic in the sense of [4] if every finitely generated subgroup of G in the sense of groups is a cyclic group.

This condition is equivalent of the following:

(i) For every $a, b \in G$ there is an element $c \in G$ and integers m and n such that $a=c^m$ and $b=c^n$.

If G is a locally cyclic group in our definition, then G is locally cyclic in the sense of [4], but the converse need not be true. In the case of locally cyclic semigroups, the analogy to (1) is not effective, that is, a locally cyclic semigroup does not necessarily satisfy the following:

(2) Every finitely generated subsemigroup is a cyclic semigroup.

However, we have

Proposition. *S is a locally cyclic semigroup if and only if every finitely generated subsemigroup of S is a subsemigroup of a cyclic subsemigroup of S.*

Definition 1. A semigroup S is said to be "power-joined" if for any two elements $a, b \in S$, there are positive integers m and n such that $a^m=b^n$.

Definition 2. A semigroup S is said to be "power-cancellative" if for any two elements $a, b \in S$, $a^m=b^m$ implies that $a=b$ where m is an arbitrary positive integer.