85. A Construction of Markov Processes by Piecing Out

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In studies of Markov processes we sometimes encounter the situations where we must piece out given Markov processes by an appropriate procedure. Examples are construction of a branching Markov process from a given Markov process which we call the non-branching part and a branching system (cf. [5], [6]), construction of a conservative Markov process from a given process of finite life time (cf. [11]), etc. In this paper we shall discuss such a procedure.

1. Notation and the main theorem. Let S be a locally compact Hausdorff space with countable base and $\bar{S}=S\cup\{\varDelta\}$ be the one-point compactification of S (if S is compact $\varDelta$ is attached as an isolated point).

At first we state the following preliminary

Lemma 1.1. *Let* $\{W, \mathcal{B}, P_x, x\in\bar{S}\}$ *be a system of probability measures on a σ-field* $\mathcal{B}$ *of* W *and let* $\mu(w, dy)$ *be a probability kernel on* $W\times\bar{S}$. *Let* $\Omega=W\times\bar{S}$, $\mathcal{F}=\mathcal{B}\otimes\mathcal{B}(\bar{S})$, *and* $\tilde{\Omega}=\prod_{j=1}^{\infty}\Omega_j$, $(\Omega_j=\Omega,\ j=1, 2,\cdots)$ *with the product σ-field* $\tilde{\mathcal{B}}=\bigotimes_{j=1}^{\infty}\mathcal{F}_j$, $(\mathcal{F}_j=\mathcal{F})$, *and put*

$$Q_x(d\omega)=P_x[dw]\mu(w, dy),$$

where we denote $\omega=(w, y)$. *Then, there exists a unique probability measure* $\tilde{P}_x, (x\in\bar{S})$ *on* $(\tilde{\Omega}, \tilde{\mathcal{B}})$ *satisfying*

$$\tilde{P}_x[d\omega^1, d\omega^2, \cdots, d\omega^n]=Q_x(d\omega^1)Q_{x_1}(d\omega^2)\cdots Q_{x_{n-1}}(d\omega^n), \tag{1.1}$$

where $\omega^j=(w_j, x_j)$.

This lemma is a consequence of Ionescu Tulcea's Theorem [7], [9].

For a given right continuous strong Markov process $\{W, x_t, \mathcal{B}_t, \zeta, \theta_t, P_x, x\in S\}$ on $\bar{S}$ with $\varDelta$ a death point,[1] we define:

Definition 1.1. A kernel $\mu(w, dy)$ defined on $W\times\bar{S}$ will be called an *instantaneous distribution* if it satisfies;

(i) For any fixed $w\in W$, $\mu(w, .)$ is a probability Borel measure on $\bar{S}$, and for any fixed Borel subset A of $\bar{S}$, $\mu(., A)$ is a $\mathcal{N}_\infty$-measurable function on W.[2]

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1) i.e. if $x_t(w)=\varDelta$ then $x_s(w)=\varDelta$ for all $s\geqq t$. We set $\zeta(w)=\inf\{t; x_t(w)=\varDelta\}$.

2) $\mathcal{N}_t=\mathcal{B}\{x_s; s\leqq t\}, 0\leqq t\leqq\infty$.