83. On Axiom Systems of Propositional Calculi. XX

By Shôtarô TANAKA

(Comm. by Kinjirô KUNUGI, M.J.A., April 12, 1966)

In their note ([1], [2]), Y. Arai and K. Iséki discuss on some theses of equivalential calculus introduced by S. Leśniewski (see, [3]).

The equivalential calculus satisfies the following two fundamental axioms:

$E1$ $EEEprEEqpErq$,

$E2$ $EEpEqrEEpqr$,

where E is the truth functor in the calculus (see, [4]).

In his paper [2], Prof. K. Iséki has given a new axiom set and has proved that the equivalential calculus is characterized by it, using some metatheorems. His results are read as below:

Lemma 1. *The equivalential calculus is characterized by*

(1) Epp,

(2) $EEpqEqp$,

(3) $EEpqEEqrEpr$.

Lemma 2. *The above axiom set is equivalent to the single axiom* $EEpqEEprErq$ (see, [2]).

In this paper, we shall also give a new axiom set of the equivalential calculus and prove that its set characterizes the equivalential calculus.

We use the two rules of inference, i.e., substitution and detachment: α and $E\alpha\beta$ imply β.

First we shall prove the following

Theorem 1. *The following axiom set, i.e.,*

1 $EEpEqrEEsqEsEpr$,

2 $EEpqEqp$,

implies the axiom set, i.e.,

(1) Epp,

(2) $EEpqEqp$,

(3) $EEpqEEqrEpr$.

For the proof we shall use prooflines by J. Lukasiewicz.

Proof. From the axioms 1 and 2, i.e.,

1 $EEpEqrEEsqEsEpr$,

2 $EEpqEqp$,

we deduce the following theses:

1 p/Erq $*C2—3$,

3 $EEsqEsEErqr$.