

81. On Axiom Systems of Propositional Calculi. XVIII

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In their notes ([1], [2]), Y. Arai and K. Iséki discuss on some theses of equivalential calculus introduced by S. Leśniewski (see, [3]).

The fundamental axioms of equivalential calculus are given by

$$E1 \quad EEEprEqpErq,$$

$$E2 \quad EEpEqrEEpqr,$$

where E is the truth functor in the calculus (see, [4]). In his paper, Y. Arai has proved that the equivalential calculus characterizes the following theses:

$$(1) \quad EEpqEqp, \quad EEEpqrEpEqr,$$

and he deduced some theses in the equivalential calculus by using the inference rule of substitution and detachment: α and $E\alpha\beta$ imply β .

In this note, we shall show that $EEpqEEprErq$, the system (1) and the set of $E1$, $E2$ are equivalent. For the proof we shall use the prooflines by J. Lukasiewicz.

Proof. From the following fundamental thesis, i.e.,

$$1 \quad EEpqEEprErq,$$

we have the following theses:

$$1 \quad p/Epq, \quad q/EEpsEsq \quad *C1 \quad r/s-2,$$

$$2 \quad EEEpqrErEEpsEsq.$$

$$2 \quad p/Epq, \quad q/r, \quad r/ErEEpsEsq \quad *C2-3,$$

$$3 \quad EEEpqsEsEpq.$$

$$3 \quad s/EEprErq \quad *C1-4,$$

$$4 \quad EEEprErqEpq.$$

$$2 \quad p/Epr, \quad q/Erq, \quad r/Epq \quad *C4-5,$$

$$5 \quad EEpqEEprEsErq.$$

$$3 \quad s/EEprEsErq \quad *C5-6,$$

$$6 \quad EEEprEsErqEpq.$$

$$6 \quad r/p, \quad s/p, \quad q/p \quad *C3q/p, \quad s/v-7.$$

$$7 \quad Epp.$$

$$1 \quad q/p \quad *C7-8,$$

$$8 \quad EEprErp.$$

$$5 \quad q/p, \quad r/q \quad *C7-9,$$

$$9 \quad EEEpqsEsEqp.$$

$$9 \quad s/EEprErq \quad *C1-10,$$

$$10 \quad EEEprErqEqp.$$