80. On Axiom Systems of Propositional Calculi. XVII

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In his note [1], K. Iséki considers the equivalential calculus, given by S. Leśniewski [2], as the abstract set $\boldsymbol{M}=\langle M, \equiv\rangle$ which satisfies the following axioms:

1 $p\equiv r.\equiv.q\equiv p:\equiv.r\equiv q$,

2 $p\equiv.q\equiv r:\equiv:p\equiv q.\equiv r$.

By a variant of Lukasiewicz symbolism we can also write these axioms as

1 $EEEprEqpErq$,

2 $EEpEqrEEpqr$,

where E corresponds to the truth functor $\equiv$ (see A. N. Prior [3]).

The purpose of our paper is to present some theses of equivalential calculus. In this calculus, we use the rule of substitution and the rule of detatchment, i.e. α and $E_{\alpha\beta}$ imply β.

Using the rules of inference we can prove from the above axioms:

1 $EEEprEqpErq$,

2 $EEpEqrEEpqr$,

the following theses:

1 p/q, q/Eqp, r/Epq $*C2$ p/q, q/p, r/q—3,

3 $EEpqEqp$.

3 $p/EpEqr$, $q/EEpqr$ $*C2$—4,

4 $EEEpqrEpEqr$.

Having proved theses 3 and 4 we are now in a position to give a proof of the following

Theorem 1. *The equivalential calculus* $\boldsymbol{M}$ *is characterized by the following axioms*:

3 $EEpqEqp$,

4 $EEEpqrEpEqr$.

Proof. We shall use prooflines by J. Lukasiewicz for the proof of theses.

3 $p/EEpqr$, $q/EpEqr$ $*C4$—5,

5 $EEpEqrEEpqr$.

5 $p/EpEqr$, q/Epq $*C5$—6,

6 $EEEpEqrEpqr$.

3 $p/EEqErpEqr$, q/p $*C6$ p/q, q/r, r/p—7,

7 $EpEEqErpEqr$.

5 $q/EqErp$, r/Eqr $*C7$—8,