73. *The Plancherel Formula for the Lorentz Group of n-th Order*

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Let $G(n)$ be the Lorentz group of n-th order, that is, the group of n-th order matrices g such that

$$^{t}gJg=J,\ \det g=1 \text{ and } g_{nn}\geqslant 1, \qquad (1)$$

with

$$J=\begin{pmatrix} 1 & & & & 0 \\ & \cdot & & & \\ & & \cdot & & \\ & & & 1 & \\ 0 & & & & -1 \end{pmatrix}.$$

In this note, we derive the Plancherel formula for $G(n)$. And we add some indications for the universal covering group $\widetilde{G}(n)$ of $G(n)$ (when $n=3$). The formula has the same form as for $G(n)$ itself.

As is well known, for an infinitely differentiable function $f(g)$ on $G(n)$ with compact carrier and an irreducible unitary representation $g\to T_g$, the operator

$$T_f=\int f(g)\,T_g d\mu(g)$$

has a trace in the corresponding representation space (here $d\mu(g)$ is a Haar measure on $G(n)$). This trace can be expressed by an invariant function $\pi(g)$ on $G(n)$ as

$$Sp(T_f)=\int f(g)\pi(g)d\mu(g).$$

This function $\pi(g)$ is called the character of the representation $g\to T_g$.

The series of irreducible unitary representations which appear in the decomposition of a regular representation (i.e. principal series) was classified in [1] and their characters was obtained in [2]. Moreover the author proved recently that the reperesentations of the Lie algebra of $G(n)$ listed in [1] exhaust all algebraically irreducible ones which are induced by completely irreducible representations of $G(n)$. Therefore the results in [1] and [2] can be considered as the results concerning all infinitesimally equivalent classes of the completely irreducible representations of $G(n)$.

With the same notations in these papers, the principal series are the continuous series: $\mathfrak{D}_{(\alpha;i\rho)}$ and, in case n is odd, the discrete series: $\mathrm{D}^{+}_{(\alpha;p)}$ and $\mathrm{D}^{-}_{(\alpha;p)}$. For $\mathfrak{D}_{(\alpha;i\rho)}$, that trace is denoted by $Sp(T^{\chi}_{f})$ with $\chi=(\alpha;i\rho)$, and the sum of the traces of $\mathrm{D}^{+}_{(\alpha;p)}$ and $\mathrm{D}^{-}_{(\alpha;p)}$ is