97. *Ideals and Homomorphisms in Some Near-Algebras*

By Sadayuki YAMAMURO

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§1. A real vector space $\mathcal{A}$ is called a *near-algebra* if, for any pair of elements f and g in $\mathcal{A}$, the product fg is defined and satisfies the following two conditions:

(1) $(fg)h=f(gh)$; (2) $(f+g)h=fh+gh$ for f, g, and h in $\mathcal{A}$.

The left distributive law: $h(f+g)=hf+hg$ is not assumed. Therefore, a near-algebra is a *near-ring* which has been defined in [6, pp. 71–74].

Let E be a real Banach space. Let f and g be mappings of E into E. We define the linear combination $\alpha f+\beta g$ (α and β are real numbers) by

$$(\alpha f+\beta g)(x)=\alpha f(x)+\beta g(x) \quad \text{for every } x\in E,$$

and the product fg by

$$(fg)(x)=f(g(x)) \quad \text{for every } x\in E.$$

Let $\mathcal{A}$ be a near-algebra of mappings of E into E. A subset I of $\mathcal{A}$ is said to be *an ideal* if it satisfies the following two conditions:

(1) *I is a linear subset of $\mathcal{A}$;*

(2) *$f\in I$, $g\in\mathcal{A}$ imply fg, $gf\in I$.*

The ideals of *distributively generated* near-rings have been studied by [2] and [3]. Obviously, near-algebras of mappings on Banach spaces are, in general, not distributively generated.

Examples (cf. [4] and [5]). 1. A mapping f of E into E is said to be *constant if*

$$f(x)=a \quad \text{for every } x\in E$$

for a fixed element $a\in E$. We denote this mapping f by c_a. Since

$$\alpha c_a+\beta c_b=c_{\alpha a+\beta b} \quad \text{and} \quad c_a c_b=c_a,$$

the set $I(E)$ of all constant mappings on E is a near-algebra. It is obvious that, if a near-algebra $\mathcal{A}$ contains $I(E)$, $I(E)$ is a minimal ideal of $\mathcal{A}$, and that $\mathcal{A}$ has no proper non-zero ideal if and only if $\mathcal{A}=I(E)$.

2. Let $\mathcal{A}$ be a near-algebra whose elements are bounded (transform every bounded set into a bounded set) and continuous mappings of E into E. Then, the set $\mathcal{A}(C)$ of compact (transform every bounded set into a compact set) and continuous mappings in $\mathcal{A}$ is an ideal of $\mathcal{A}$.

§2. Let I be an ideal of a near-algebra $\mathcal{A}$. Let us write