138. Γ-Bundles and Almost Γ-Structures. II

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In [2], the author introduce the notion of almost Γ-structure and give an integrability condition of almost Γ-structures. It suggests us that there seems to be useful that to use some differential geometric aspects of tangent microbundles in the study of Γ-structures. In this note, we treat pseudoconnections of topological microbundles which was defined in [2], and show the following theorem: There is a 1 to 1 correspondence between the set of equivalence classes of Γ-structures on X and the set of Γ-equivalence classes of pseudoflat Γ-bundle structures of the diagram $X \xrightarrow{\Delta} X \times X \xrightarrow{p} X$, where Δ is the diagonal map and p is the projection to the first component (cf. [4]). Notations of this note are similar that of [1], [2].

1. *Pseudoconnection of topological microbundles.* For an element a of $\boldsymbol{R}^n$, we define the parallel transformation t_a by

$$t_a(b)=b-a,\ b \in \boldsymbol{R}^n.$$

Lemma 1. *A homeomorphism f from a neighborhood of the origin of $\boldsymbol{R}^n$ into $\boldsymbol{R}^n$ is a parallel transformation if and only if*

$$(df)(a, b)=b-a,$$

where $(df)(a, b)=f(b)-f(a)$.

On the other hand, if $\alpha_U \in C^1(U, \boldsymbol{R}^n)$, then

(1) $$\delta(t_{\alpha_U})(x_0, x_1, x_2)=t_{(\delta\alpha_U)(x_0,x_1,x_2)},$$

where the multiplication is defined to be the compositions of $\boldsymbol{R}^n$ and $\delta(t_{\alpha_U})$ and $(\delta\alpha_U)$ are given by

$$\delta(t_{\alpha_U})(x_0, x_1, x_2)=t_{\alpha_U(x_1,x_2)}t_{\alpha_U(x_0,x_2)}{}^{-1}t_{\alpha_U(x_0,x_1)},$$

$$(\delta\alpha_U)(x_0, x_1, x_2)=\alpha_U(x_1, x_2)-\alpha_U(x_0, x_2)+\alpha_U(x_0, x_1).$$

Definition. Let $\{\varphi_{UV}(x)\}$ be a transition function of an n-dimensional topological microbundle ξ over normal paracompact topological space X, then a collection $\{t_{\alpha_U(x,y)}\}$, $\alpha_U(x, y) \in C^1(U, \boldsymbol{R}^n)$, is called a pseudoconnection of $\{\varphi_{UV}(x)\}$ if $\{t_{\alpha_U}\}$ satisfies

(2) $$\varphi_{UV}(x)^{-1}t_{\alpha_U(x,y)}\varphi_{UV}(y)=t_{\alpha_V(x,y)}.$$

According to [1], we call the collection $\{\delta(t_{\alpha_U})\}$ to be the curvature form of $\{t_{\alpha_U}\}$.

Definition. We call $\{t_{\alpha_U}\}$ is a flat pseudoconnection if the curvature form $\{\delta(t_{\alpha_U})\}$ of $\{t_{\alpha_U}\}$ is equal to 0.

Definition. $\{\varphi_{UV}(x)\}$ is called a pseudoflat microbundle if $\{\varphi_{UV}(x)\}$ has a flat pseudoconnection.