134. Operators of Discrete Analytic Functions and Their Application

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(Comm. by Kinjirô KUNUGI, M.J.A., June 13, 1966)

Introduction. In the previous paper [4] we have studied basic properties of operators of discrete analytic functions. In this paper we shall study the uniform convergence of sequences of operators and show operational solutions of a discrete Volterra integral equation and a linear discrete derivative equation by making use of operators of discrete analytic functions.

1. Uniform convergence of sequences of operators. The set A of all discrete analytic functions is a linear space of infinite dimension. By the norm $\|f\|$ of $f\in A$ we understand the number $\|f\|=sup\,|f(x, y)|$, where (x, y) is a finite lattice point in the first quadrant.

By the norm, the *uniform convergence* in A is defined as follows: A sequence f_n of A converges uniformly to an element f of A if and only if the sequence $\|f_n-f\|$ tends to 0 as $n\to\infty$. The convergence is denoted by

$$\lim_{A} f_n=f.$$

The normed space A is *complete*, i.e. any Cauchy sequence is convergent.

Thus A is a *Banach space.*

Theorem 1.1. *If f_n and $g_n\in A$, and $\lim_{A} f_n=f$, $\lim_{A} g_n=g$, then $\lim_{A}(f_n*g_n)=f*g$.*

This means that $*$ is *continuous* in the *norm topology.*

A sequence of operators a_n is said to be *convergent in* ***Op***, if divided by a suitably chosen operator q, it becomes a sequence of functions $\in A$ uniformly convergent to $f\in A$. Then we have

$$\lim_{Op} a_n=q\lim_{A}\left(\frac{a_n}{q}\right). \tag{1.1}$$

Theorem 1.2. *If $\lim_{Op} a_n=a$, $\lim_{Op} b_n=b$, then*

$$\lim_{Op}(a_n\pm b_n)=a\pm b,\ \lim_{Op}(a_nb_n)=ab. \tag{1.2}$$

Theorem 1.3. *Let a be a complex number. The power series*

$$\sum_{n=0}^{\infty}{}' \frac{a^n z^{(n)}}{n!} \tag{1.3}$$

converges uniformly in any bounded domain in the first quadrant.