133. *Operators of Discrete Analytic Functions*

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1. The convolution product. We are concerned with complex-valued functions $f(x, y)$ of two independent integral variables x and y satisfying the following condition.

Let x and y be any integers, and put

$f_0=f(x, y)$, $f_1=f(x+1, y)$, $f_2=f(x+1, y+1)$, $f_3=f(x, y+1)$,
$\bar{f}_0=(f_0+f_1)/2$, $\bar{f}_1=(f_1+f_2)/2$, $\bar{f}_2=(f_2+f_3)/2$, $\bar{f}_3=(f_3+f_0)/2$.

Let $p(\neq 1)$ is an arbitrary real or complex number, then

$$\bar{f}_2-\bar{f}_0=p(\bar{f}_1-\bar{f}_3)$$

is equivalent to

$$L_q f\equiv f_0+qf_1-f_2-qf_3=0, \tag{1.1}$$

where $q=(1+p)/(1-p)$.

The function f is said to be *discrete analytic* in R, if the condition (1.1) is satisfied for every x and y in a simply connected region R in the x-y plane. The set of all discrete analytic functions in R is denoted by $\boldsymbol{A}(R)$, or briefly $\boldsymbol{A}$. Duffin's discrete analytic functions [1], [2] are the special case whence $p=q=i$.

Denote for brevity

$$f(x, y)\equiv f(z),\ z\equiv(x, y),\ z_r\equiv(x_r, y_r)$$

where x_r and y_r are integers. The points of the x-y plane with integer coordinates are called *lattice points*.

Let z_r, z_{r+1} be consecutive lattice points. The *double dot integral* along a chain $z_0, \cdots, z_r, z_{r+1}, \cdots, z_n$ is defined by

$$\int_{z_0}^{z_n} f(t)\colon g(t)\delta t\equiv\sum_{r=0}^{n-1}\bar{f}_r\bar{g}_r\delta_r,\quad \delta_r=\pm 1,\ \pm p, \tag{1.2}$$

where $\bar{f}_r=[f(z_r)+f(z_{r+1})]/2$, $\bar{g}_r=[g(z_r)+g(z_{r+1})]/2$, $\delta_r=1$ or -1 respectively if $y_{r+1}=y_r$ and $x_{r+1}=x_r+1$ or $x_{r+1}=x_r-1$, and $\delta_r=p$ or $-p$ respectively if $x_{r+1}=x_r$ and $y_{r+1}=y_r+1$ or $y_{r+1}=y_r-1$.

The double dot integral of two integral variables

$$\int_0^z f(z-t)\colon g(t)\delta t$$

is said the *convolution product* of $f(x, y)$ and $g(x, y)$, and is denoted by $f*g$, i.e.

$$(f*g)(z)\equiv\int_0^z f(z-t)\colon g(t)\delta t, \tag{1.3}$$

where $0=(0, 0)$ and $z=(x, y)$.

Equation (1.3) requires that not only the chain $0=z_0, z_1, \cdots, z_n=z$ lies in R, but also the chain $z-z_0, z-z_1\cdots, z-z_n$ lies in R.